Phys 2425

University Physics Workbook Part I

Calculus based course, Tyler Junior College, Summer I 2013

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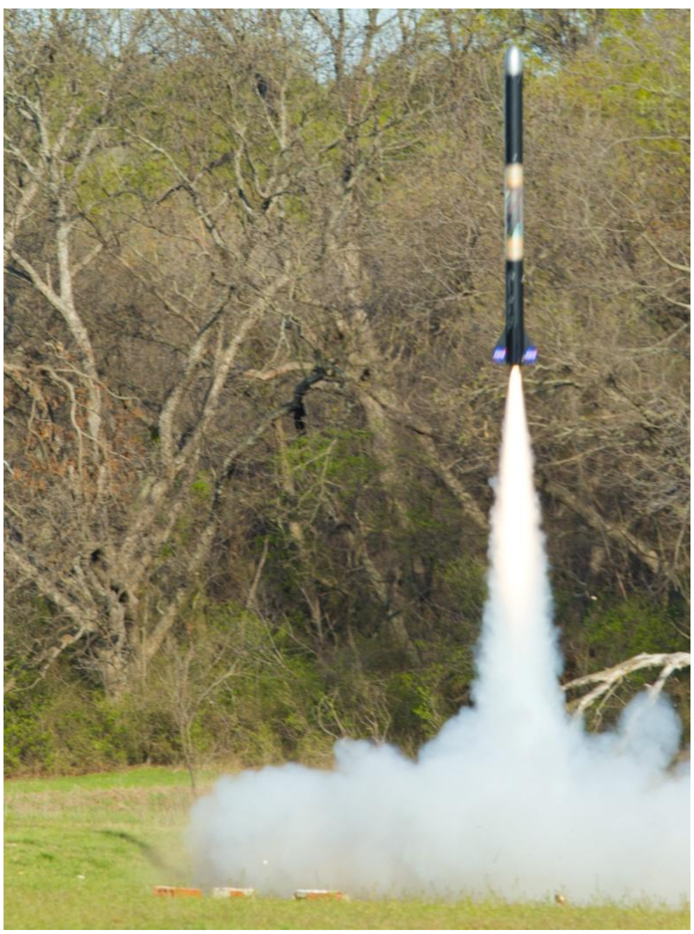
Tyler Junior College



Acknowledgements: This workbook has been developed over a number of years by numerous collaborators whose names have been lost and forgotten. Our thanks go to those unsung heroes who have contributed to this work. Portions of this work are used by permission and/or fair use of Dr. Bob Abel (Olympic College), M. Brooks, G. Sherman, M. Broyles, A. Kumar (Collin College), The Science Source, Cenco Physics, Vernier, and AAPT.

Tareleton State College in Stephenville, TX, Rocket Launch on 4/5/2013. Photo by Jim Sizemore.

Hyperlinks work, primary web site at http://iteach.org/funphysicist/. Offsite files at:
PDF: http://funphysicist.weebly.com/uploads/2/0/3/8/20383539/calc_physics_workbook_v1.pdf
Word: http://funphysicist.weebly.com/uploads/2/0/3/8/20383539/calc_physics_workbook_v1.docx

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- Collate
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- Do not Hole Punch
- Spiral Binding
- No to Colored Paper – plain white
- No to Color printing
- Deliver to bookstore

Special Instructions

1. **PLEASE MAKE EVERY EFFORT TO DELIVER THIS TO THE BOOKSTORE BY THE FIRST DAY OF CLASS**
2. **BINDING ON LEFT SIDE LIKE A REGULAR BOOK – NOT ON RIGHT**
3. Print in black & white
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5. **CANARY** cardstock front and back.
6. Print so odd numbered pages are on the right.
7. Deliver one copy to <your name>, <your office> (<your mailroom>)

TJC Bookstore Information
TJC Course Name: University Physics I
TJC Course Number: PHYS 2425
Instructor's Name: <your name>
Bookstore Up-Charge (if known): $5.00

Table of Contents

Profits from the sale of this lab manual will go toward student activities and professional development.

University Physics Workbook Part I
Lab Report Guidelines

Lab Supplies

1. Ruler with centimeter scale
2. Protractor
3. Pencil
4. CALCULATOR
5. Graph Paper

Student's Lab Responsibility

1. Be on time!
2. Study lab before class – PRE-LAB ASSIGNMENTS WILL BE REQUIRED! For example, your instructor may need to approve your data collection prior to class.
3. Actively participate as an individual in a group.
4. Be careful with equipment.
5. Feel free to move around and talk in lab, but <u>DO NOT DISTURB OTHERS!</u>
6. When you leave the lab, *return equipment precisely where you got it from*, place chairs under tables, and clean your work area.

Links to expanded discussion:

The following is about 24 pages worth of how-to for scientific writing with links to additional resources. I've attempted to condense this and more to 8 pages, however this is a good discussion – a little light reading.

IMPORTANT NOTE: Links are active in the online version of this lab manual.

pdf version:
http://writingcenter.unc.edu/resources/handouts-demos/pdfs/Scientific%20Reports.pdf

html version:
http://writingcenter.unc.edu/resources/handouts-demos/specific-writing-assignments/scientific-reports

Guidelines for Lab Reports

<u>*At a very basic level a lab report expresses clear thinking about a topic under investigation. Your goal is to think, investigate, and express your investigation clearly!*</u>

We are practicing writing a research report to prepare you for your future work. No matter what you do in the future you will be called upon to write reports, unless you're content to flip burgers for the rest of your life or something equally mundane. Our practice contributes to the knowledge and skills your future will require. Classroom thinking is, "I'm doing this because the teacher requires it." **Broader thinking is, "I'm practicing writing skills that my future endeavors will require."**

Think of your audience. People who read research reports are interested in two things, (1) what is the information contained in the report, and (2) are the findings valid and legitimate. Write your report to answer these two basic questions.

A lab report generally follows the scientific method, that is, (1) research, (2) make a hypothesis, (3) design an experiment, (4) perform the experiment, (5) analyze the experiment to determine if it confirms or contradicts the original hypothesis, and (6) report your findings. Your goal is to clearly, completely, and yet concisely explain how you followed the scientific method in performing your experiment.

A key prerequisite to a good report is to know and understand the scientific principles underlying your experiment and why and how the experiment tests those principles. IT IS, therefore, IMPERATIVE TO READ AND UNDERSTAND THE LAB INSTRUCTIONS BEFORE COMING TO LAB. Understand the following:

1. What you are going to do, that is, what's the procedure?
2. Why are we going to do it that way?
3. What are we hoping to learn from this experiment?
4. Why would we benefit from this knowledge?

Answering these questions lead you to a more complete understanding of the experiment think about the "big picture" leading to a better lab report.

Ask questions of the lab instructor. If you don't know an answer the instructor can help explain it or, at least, help you figure it out.

Before the Lab

A. READ AND UNDERSTAND THE LAB INSTRUCTIONS!
B. Plan the steps of the experiment carefully with your lab partners.
C. Design a table to record your data.
D. Assign each member of the lab group a "job" and rotate that job each lab. Don't have somebody doing the same thing all semester.
E. EVERYBODY does the experiment. Have one person do one measurement, a second person do the second measurement, a third person the third measurement, etc.

During the Lab

F. All members record the data in your personal lab book.
G. Record data carefully in a well organized manner.
H. Consult with your lab partners as you are performing work.

Lab Reports

I. DO YOUR OWN WORK! You may discuss this with your lab partners, however copying will result in a zero grade, or worse, for all persons in the lab group. Copying is a violation of academic ethics and punishment may be severe. Students should adhere to high ethical standards.
J. The instructor may, optionally, require reports to be submitted as a lab group. Consult your instructor or their syllabus regarding this.
K. Lab reports are due *at the beginning* of the next scheduled lab, however write a first draft prior to leaving. If you have forgotten something then you can acquire what you need. YOUR INSTRUCTOR MAY REQUIRE A REVIEW OF DATA AND CALCULATIONS PRIOR TO LEAVING LAB OR MAY REQUIRE YOU TO TURN IN YOU LAB NOTEBOOKS.
L. Neatness, readability, and a well-organized report is the primary requirement.
M. The prose, tables, and equations of your report must be typed.
N. Graphs and drawings may be done using pencil **and straight edge**, but grids must be included and it must be to scale. Be sure to include them in the proper order. YOUR INSTRUCTOR MAY REQUIRE HAND DRAWN GRAPHS – you need to learn to walk before you can run, you need to be able to create hand drawn graphs before you can create computer graphs, you may know programs insides and out, but don't know how to apply them to math, science, and technology problems.

O. Use the following outline format and **do not deviate from this order** when writing your lab report. You may mix typed and hand written information, however DO NOT, for example, staple raw data to the end of the report.

Course Name and Section
Experiment Name
Date experiment was performed
Lab Partner's Names and Duties
Title

I.	Purpose	A few sentences **stating the quantitative hypotheses you will be testing**.
II.	Theory/Introduction	Discuss key equations
III.	Procedure Title Procedure Step Use subtopic headings as an aid to organizing your information Next Procedure Step etc.	USE DIAGRAMS!
IV.	Data	ORGANIZE IN TABLES!
V.	Graphs	Hand drawn using a straight edge.
VI.	Sample Calculations	Provide a sample or all calculations used in the lab
VII.	Results	1. These are calculations on the data. 2. ORGANIZE IN TABLES! 3. ALWAYS REPORT THEORETICAL EXPECTATIONS! 4. ALWAYS REPORT A %ERR OR %DIFF! 5. Show the step-by-step reasoning leading to your conclusion.
VIII.	Conclusions	**State your step-by-step reasoning and conclusion about each hypothesis tested** and why you believe your conclusion is reasonable.
IX.	Questions	Answer any questions posed in the lab instructions

DO NOT DISMANTLE THE APPARATUS UNTIL AFTER YOUR INSTRUCTOR HAS REVIEWED (SIGNED OFF) ON YOUR DATA AND CALCULATIONS. It may be necessary to repeat portions of your experiment.

P. Be consistent with your outline *format* throughout the entire report.
Q. Do not write on the back of your paper.
R. Organize your report so that there are no large gaps between topic headings.

S. **General Guidelines:**
1. Use **correct spelling, grammar** and complete sentences to express your ideas. Word processors are very good at this.
2. **Be complete and clear, yet concise.** Hack writers typically bloat their bloat their narratives with excessive and repetitious wordiness. Unfortunately in school you are required to write an n-page report which encourages this. It's

time to break that habit and write with meaning and clarity – not merely write to fill space.
3. Avoid repetition.
4. Be consistent with your terminology. If a word or phrase was used in one part of your report, use the same word or phrase for the entire report.
5. Carefully prepare your report.
6. Make your report easy to follow.
7. Be specific.
8. Explain your terms – even if they were in the lab instructions.
9. Avoid jargon.
10. Pretend you're writing for a student in another section of this course and write in a way that student may understand your work.

T. **Title:**
1. Is the title brief yet clear enough to identify the experiment?
2. Does it express all the features of interest?

U. **Purpose:**
1. The **purpose and conclusion are the most important parts** of you reports.
2. State the **hypotheses you will be quantitatively testing**.
3. As you write the remainder of you report make sure you **show the step-by-step reasoning processing leading to your conclusions for each and every hypothesis** you are testing.
4. What is your testable hypothesis?
   a. *Not a hypothesis:* There is significant relationship between the temperature of a solvent and the rate at which a solute dissolves.
   b. *Hypothesis:* As temperature increases the rate at which a solute dissolves increases.
5. A purpose goes one step further; for example, determine the mathematical relationship between the temperature of the solvent and rate of solute dissolution.
6. What leads you to believe your hypothesis is supported by evidence? Even outside-the-lab experience may be used. For example, you note that sugar seems to dissolve faster in hot water than in cold.
7. Justify the experimental approach to test your hypothesis.
8. What is the rationale of the experiment as it relates to your hypothesis?
9. In a short paragraph list with short discussion (phrase or sentence) the theories you intend to confirm – do not list equations.
10. Insure your list of theories or questions investigated are complete.
11. What is the experimental objective? Why is it important to do this experiment?
12. Think about what the experiment is testing and list the theories being tested *even if not mentioned in the lab manual.*
13. Indicate the main topics to be tested.
14. Outline the purpose, scope, and approach of the experiment.
15. Brevity is important. Mention or list items – don't thoroughly discuss them. You'll discuss them completely in the next section.
16. Are there details which could be reduced, put in an appendix, or omitted?

V. **Theory/Introduction:**

1. Demonstrate that you understand the context for this experiment. Using the dissolution rate of a solute as has been our example, you may recall lecture discussions about polar molecules that motivate your hypothesis.
2. List equations pertaining to theories being tested, the theories mentioned in the purpose, and thoroughly (yet concisely and clearly) discuss those theories and equations.
3. Discuss how your experiments will confirm or deny those theories and what error analysis you will perform.
4. Define symbols and abbreviations the first time they are used.
5. Discuss the applicability of prior work with adequate references.
6. Your experimental approach needs to be adequate. In general, attempt experiments at the extremes of the capabilities of the instrument and at least one attempt in the middle. In general this will require at least three measurements and spread measurements out evenly.

W. **Procedure:**

1. You must describe your procedure in sufficient detail that your experiment may be reproduced.
2. Be precise, but stay relevant. Ask yourself, “Would it make a difference if a part were a different size or different material?”
3. Provide enough details to prevent the experiment from going awry if someone else tries to perform it.
4. Explain your rationale. If you capped a test tube after adding solute, why did you do that?
5. What is your control experiment? Did other researchers obtain specific results performing the experiment under specific conditions? You may repeat this as a control. Or, are you comparing your results to an existing theory?
6. Describe the steps like a story in chronological order. Especially if order is important to the procedure, present the steps in order.
7. Don’t use the recipe approach. Don’t, for example, specify amounts such as 50 ml. Instead state, “Measure the water used in the experiment, record this in the data table, and add it to the beaker.”
8. Usually we want to use past tense and third person reporting. Since first person is usually more readable, at this stage, we won’t be particular about this. Remember, however, in the future this distinction may be important.

X. **Diagrams:**

1. Draw diagrams neatly using a straightedge.
2. Identify the equipment used and label the parts. Include them in your report in the correct order.

Y. **Data Tables:**

1. ALWAYS USE TABLES! The strength of a table is the ability to supply large amounts of exact data.
2. NEVER REPORT DATA IN PARAGRAPH FORM!
3. Use data tables to identify and organize your data and information.
4. Choose a good clear descriptive title for your table.
5. Number your table.
6. Arrange the table so readers read vertically, not horizontally. Have a header row and then data in columns.

7. Center numbers in a column or line up on the decimal point.
8. Be sure to identify the data collected with the appropriate units of measurement in these tables.
9. Use clear descriptive captions that are easily identifiable.
10. Round to 3 sig. figs. or more such that the final result is accurate to 3 sig. figs.
11. If you have few measurements and few calculations, sometimes it makes more sense to report data and results together. This is acceptable, however if neatness, organization, and clarity are improved, report data and results separately.

Z. **Graphs:**

1. Most of the time data is in the form of a response variable as a function of the independent variable. Data in this form is plotted in a graph. Always graph data if it is in this form.
2. The strength of a graph is the ability to dramatically illustrate trends. Note the strength of tables is presentation of exact information. A graph trades exactness for illustrative ability. A graph helps readers better understand your results.
3. A beginner problem is not making graphs big enough. **Use an entire page** for your graphs AND make sure your data consumes most of the page.
4. What is the message the graph is attempting to convey?
5. Identify the graph with a clear **descriptive title**.
6. Number your graph.
7. **Use Graph Paper**.
8. Use a French curve or ruler to connect the dots or mark the best fit line.
9. Identify the **quantity being graphed along each axis** (force, distance, etc.)
10. Identify the **unit of measurement** of each quantity graphed.
11. The independent variable (what you set) goes on the horizontal axis and the dependent variable (what your measure) goes on the vertical axis.
12. Clearly mark the scale of measurement along the x- and y- axes.
13. Insure the **scale is uniform**.
14. Start the **scale from (0,0)**. Choose scales of 1, 2, 5, 10, 20, 50, 100, 500, or 1000 units per division. Do not, for example, use units of 3, 7, 11, etc. units per division.
15. Write the title, quantity, measurement information, and units in the margin - NOT INSIDE THE GRAPH GRID.
16. Denote the experimentally obtained data on the graph by small, precise dots.
17. Keep it simple. Draw three separate graphs, for example, rather than three overlying and confusing lines on a solitary graph.
18. Ask yourself if the **best-fit line must go through the origin or not**?
19. If more than one graph is placed on a grid, be sure to CLEARLY identify each curve along with the scale of measurement that applies to that curve, e.g. color code and key.

AA. **Sample Calculations:**

1. Write all the general formulae you are applying, and then give an example calculation for each formula using experimental data. Show only a single example for each formula.
2. Indicate the units on your calculations
3. Calculate percent difference or error.
4. Use lined paper and show only one calculation per line.

5. ALLOW ADEQUATE SPACE BETWEEN CALCULATIONS SO THAT THEY CAN BE EASILY READ!

BB. **Results:**

1. **Summarize** data with calculated averages, slope of line, % Err, % Dev, etc.
2. **ALWAYS TABULATE** if possible. Don't write, for example, "R_1 was 50 Ω, R_2 was 50 Ω, measured R_s was 101 Ω, the calculated R_s was 100 Ω, and the % Err was 1.00%. For the second experiment, R_1 was 40 Ω, R_2 was 60 Ω, measured R_s was 98 Ω, the calculated R_s was 100 Ω, and the % Err was 2.00%, …" Use tables instead as shown in the following example:

R_1 (Ω)	R_2 (Ω)	Theory $R_s = R_1 + R_2$ (Ω)	Measured R_s (Ω)	% Err
50	50	100	101	1%
40	60	100	98	2%
. . .	. . .	. . .	. . .	. . .

3. Create your own well organized tables. This is the art form of learning to better report scientific information and you only learn by doing.
4. If possible, **graph your tabulated information**. Some reviewers demand that you do not replicate tabulated information in a graph and vice-versa. That is not the case for our college's labs and especially at the student's level of math skills. Tabulate the information you are graphing.
5. Write a clear descriptive summary for each table and graph.
6. **Always** report % Err or % Diff (whichever is applicable)
7. **Always** compare experiment to any and all applicable theories even if not specified in the objective or purpose statement.
8. Be clear and complete, yet concise.
9. **Most important:** How do your results confirm or contradict your original hypotheses?
10. Do you observe trends?
11. Do not make conclusions.

CC. **Conclusion/Discussion:**

1. This is the most important part of the report. In this section you will **summarize your purpose, the hypotheses tested, argue the validity of the methods used to confirm or contradict the hypotheses, did you confirm or contradict your hypotheses, and the implications** of your findings.
2. Use your best communication skills to convey your message. If reasoning is difficult to follow it detracts from the report.
3. Write in paragraph form using complete sentences.
4. Review the purpose of the experiment to help you formulate your conclusions. Paraphrase a restatement of the purpose.
5. Explain whether the data supports or contradicts the hypotheses and support this with your evidence.
6. Be clear and complete, yet concise. The extremes of too much brevity and excessive verbosity are to be avoided.

7. Facts, arguments, and conclusions need to be technically valid and accurate.
8. Avoid bias and guesswork.
9. Acknowledge anomalous data, or deviation, from expectations.
10. If you should have done a better experiment, be honest, however since your results are reviewed before completion of the lab we should not encounter this. You will be penalized for incomplete labs and doubly penalized for dishonesty.
11. If your experiment had a weakness, be precise about that weakness, why and how that weakness affected your data, and what you would do to eliminate that weakness. Avoid lame excuses like human error, it was the second Tuesday of the month, etc.
12. What was this experiment about, what are the findings and implications, and why do they matter?
13. From your results, **connect the dots** clearly, completely, and concisely **leading to your conclusion about all hypotheses tested**.
14. Relate your findings to previous work including lecture discussions. Present your work in context.
15. Do your results support or not support your expectations (theory)?
16. Include possible reasons for any percent difference or error obtained in the experiment.
17. Lastly, what are the implications of this work.

DD. **Questions:**

1. Answer ALL the questions raised in the purpose, introduction, and theory/introduction sections.
2. **Restate the question.**
3. THIS IS NOT A SUBSTITUTE FOR A CONCLUSION!
4. Explain your reasoning thoroughly including using the SOLVE Method (Appendix B).
5. Use your best communication skills to convey your message. If reasoning is difficult to follow it detracts from the report.
6. Write in paragraph form using complete sentences.
7. Be clear and complete, yet concise. The extremes of too much brevity and excessive verbosity are to be avoided.
8. Facts, arguments, and conclusions need to be technically valid and accurate.

Lab Grades

1. Lab reports are evaluated on content, technical validity, organization, and presentation of information and neatness. <u>This reflects the quality of your work!</u>
2. I will end with the statement I started with: **At a very basic level a lab report expresses clear thinking about a topic under investigation. Your goal is to think, investigate, and express your investigation clearly!**

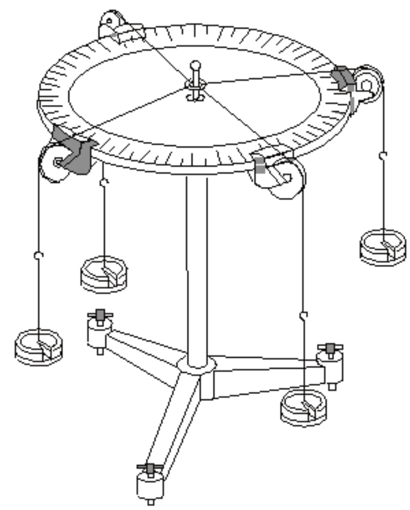
University Physics Workbook Part I
Force Table (Vectors)

Forces are an important part of physics and the world around us. Forces keep our feet on the ground, help us to move around the planet, and allow us to do other wonderful things – like ride a bike. Forces are a defined in physics as a vector quantity. That is to say they have a magnitude and a direction. In this lab you will study forces and, hopefully, better understand force and equilibrium (when forces sum to zero) concepts. Note: When the purpose says something like "investigate," "study," etc., it's actually saying to test the theory – confirm whether or not it agrees with experiments.

Purpose:

To experiment with the treatment of forces as vectors.

Procedure:



According to Newton's first law of motion, a body in motion will maintain its condition of motion unless acted upon by unequal forces. Therefore, if an object is in equilibrium, the vector sum of the forces acting on the object must be zero.

The body in our experiment will be the ring at the center of a force table. We will produce forces on the ring by hanging masses from it in various directions. The force which will produce equilibrium (called the equilibrant force) shall be determined experimentally and compared with that determined analytically by treating force as a vector. Equilibrium, is achieved when the ring is centered on the force table.

Due to the presence of friction in the pulleys, small amounts of mass could be added or subtracted from the equilibrant measurement and small changes in pulley's position are possible without any noticeable effect; the ring does not move. This will be a source of error and should be addressed in your conclusion.

Find experimentally the mass which must be added to produce equilibrium for each of the following cases and determine its direction as outlined above:

1. 500g at 0°, 300g at 90°
2. 400g at 0°, 600g at 140°
3. 300g at 0°, 400g at 120°, 600g at 310°

Then, work each problem mathematically to determine the equilibrant force. Work one problem per page. For each problem, neatly construct a force diagram showing the given force vectors, the measured equilibrant, and the calculated resultant. Show all work in an orderly and logical progression. Find the percent error in each case by using the following formula:

$$\%error = \left|\frac{calculated - measured}{calculated}\right| x\ 100$$

University Physics Workbook Part I
Free Fall (Constant Acceleration)

Free Fall in a Gravitational Field

All things must fall to the Earth. We know this is due to gravity and we know that it happens in about the same manner all over the world. But, how fast do we fall? If we assume that the resistance of the air around us is negligible, how fast would some object fall under the influence of gravity?

Purpose:

Show that the acceleration due to gravity near the surface of the Earth is constant and to determine its value.

Procedure:

Your instructor has set up the Behr Free-Fall apparatus and demonstrated its use. Using the high-voltage spark timer and the spark-recording tape, make a record of the position of the falling plummet at 1/60 second intervals for at least twenty intervals. Measure the position of each spark dot using a meter stick. Record this data under the position column in the data table.

Some groups will "beta" test two alternative procedures. The first is using the Vernier sonic position sensor to record distance and time. The second is to drop balls from known heights and record distance and time. We are in the process of determining the best labs for future semesters.

Subtract each of the position readings from the position reading immediately preceding it to find Δx. Record Δx_1, Δx_2, Δx_3, etc. ($\Delta x_1 = x_3 - x_2$).

Divide the Δx intervals by the 1/60 second time interval to calculate the velocity of each of the Δx intervals. $\bar{v} = {}^{\Delta x}/_{\Delta t}$. Record these values in the data table.

Graphical Analysis:

Plot a velocity vs. time graph. The slope of the velocity-time graph, ${}^{\Delta v}/_{\Delta t}$, is equal to the acceleration of the falling plummet. Assuming the acceleration due to gravity near the surface of the Earth to be 9.80 m/s^2, find the percent error.

Plot a position vs. time graph. The slope of the position-time graph, $\frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$, represents the velocity of the plummet at a given instant in time. The instantaneous velocity is determined by finding the slope of the tangent to the curve at a given instant. Determine graphically from your position-time graph the velocity of the plummet at the 11.5 time interval.

Compare the calculated value of the velocity at the 11.5 sec. instant with the velocity recorded in the data table during the 12th time interval. The recorded velocity, being found by $\frac{\Delta x_{12}}{1/60}$, is the velocity for the interval and should approximate the instantaneous velocity at the 11.5 instant.

From your data, determine the velocity at the 11.5 sec. time interval by finding the midpoint of that velocity time interval. Compare the graphically determined velocity to the velocity from you data for the 11.5 sec. time interval and find the percent difference.

Data Table:

Reading	Time (s)	Position (m)	Δx (m)	V (m/s)
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				
16				
17				
18				
19				
20				

University Physics Workbook Part I
Projectile Motion (Motion in 2 Dimensions)

To predict where a ball will land on the floor when it is shot off a table at some angle above the horizontal, it is necessary to first determine the initial speed (muzzle velocity) of the ball. This can be determined by shooting the ball horizontally off the table and measuring the vertical and horizontal distances through which the ball travels. Then, the initial velocity can be used to calculate where the ball will land when the ball is shot at an angle.

Purpose:

To predict and verify the range of a ball launched at an angle. The initial velocity of the ball is determined by shooting it horizontally and measuring the range and the height of the launcher.

Materials:

C-Clamp
Projectile Launcher
Plumb - Bob
Target Paper with Carbon Paper
Projectile
Meter Stick

Some Preliminary Background Information:

Horizontal Initial Velocity

For a ball shot horizontally off a table with an initial speed, v_0, the horizontal distance traveled by the ball is given by $x = v_0 t$, where t is the time the ball is in the air. Air friction is assumed to be negligible.
The vertical distance the ball drops in time t is given by $y = \frac{1}{2}gt^2$.
The initial velocity of the ball can be determined by measuring x and y.

Initial Velocity At An Angle

To predict the range, x, of a ball shot off with an initial velocity at an angle, θ, above the horizontal, first predict the time of flight using the equation for the vertical motion:

$$y = y_0 + (v_0 \sin\theta)t + \frac{1}{2}gt^2$$

Where y_0 is the initial height of the ball and y is the position of the ball when it hits the floor. Then use $x = (v_0 \cos\theta)t$ to find the range.

Procedure:

Initial Velocity of the Ball

Put the plastic ball into the projectile launcher and cock it to the long-range position. Fire one shot to locate where the ball hits the floor. At this position, tape a piece of white paper to the floor. Place a piece of carbon paper (carbon side down) on top of this paper. Do not tape it down. When the ball hits the floor, it will leave a mark on the white paper. Remove the tape when the experiment is complete.

Fire at least five shots (for more uniform results, pull the string perpendicular to the launcher.)

Measure the vertical distance from the bottom of the ball as it leaves the barrel (this position is marked on the side of the barrel) to the floor. Record this distance as y in your data table.

Use a plumb – bob to find the point on the floor that is directly beneath the release point on the barrel. Measure the horizontal distance along the floor from the release point to the point that you determine to be the average of the points at which the ball hits the floor. Record this as the horizontal distance, x, in your data table.

Using the vertical distance and the horizontal distance, calculate the time of flight and the initial velocity of the ball. Repeat this procedure for the short range and the medium range positions.

Range of the Ball Shot at an Angle

Adjust the angle of the projectile launcher to 60^0.

Shoot the ball at least ten times with each of the velocities, v_s, v_m, and v_l.

Choose a single point to represent the average position of the projectile when shot at v_s, v_m, and v_l.

Using the initial velocities, v_s, v_m, and v_l, found in the first part of this experiment and the vertical heights from which the ball is shot, calculate the new times of flight and horizontal distances.

Abbreviated Data Tables – does not include individual points. What does your data table need to look like to collect individual data point, calculate mean, and calculate errors?:

Table A: Determining Initial Velocity

Launch Position	y (m)	x (m)	v (m/s)
Short			
Medium			
Long			

Table B: Confirming the Predicted Range

Trials	Height (m)	Launch Angle (θ)	Initial Velocity (m/s)	Time of Flight Cal. (s)	Range, x_c cal. (m)	Range, x_e exp. (m)	% Diff.
Short							
Medium							
Long							

University Physics Workbook Part I
Friction (Inclined Plane)

Frictional resistance to the relative motion of two solid objects is usually proportional to the force which presses the surfaces together as well as the roughness of the surfaces. Since it is the force perpendicular or "normal" to the surfaces which affects the frictional resistance, this force is typically called the "normal force" and designated by N.

Purpose:
To determine the static and kinetic coefficients of friction between a wooden block and an inclined plane.

Materials:

Mass set	Pulley
Wooden block	Triple beam balance
Inclined plane	string

Some Preliminary Questions:
Draw a free-body diagram of the system. From the diagram, can you see a relationship between the block and the hanging mass?

Procedure:
<u>Part I: Static and Kinetic Friction</u>
When the block is placed on an inclined plane, a force $\mathbf{F_d}$, acts down the plane trying to pull the block down the plane, but a frictional force, f, opposes $\mathbf{F_d}$. However, if the angle $\boldsymbol{\theta}$ that the incline makes with the horizontal is increased, the block will eventually slide down the plane. When this occurs, we can write the following equation:

$$F_d = f$$

We are concerned with two frictional forces – the static frictional force, $\mathbf{f_s}$, which tends to keep the block from moving from rest and the kinetic frictional force, $\mathbf{f_k}$, which tends to retard the motion of the block after it has started moving.

<u>Static Friction:</u>
If the block is placed on the inclined plane and the angle θ is increased slowly, the block will eventually move. Thus

$$F_d = f_s \qquad \text{Equation 1}$$

But what are these two forces? Can we break them down into simpler terms that are familiar to us – like mass and gravity? The answer of course is yes. Recalling from Newton's second law; force is equal to the mass times the acceleration of that mass – in this case gravity. So we can write the following:

$$F_d = Mgsin\theta_s \qquad \text{Equation 2}$$

Where we have taken into account the angle of the incline. If you draw a free-body diagram, you can see that this is part of the normal force acting on the block - the y-component to be exact. This leads us to the conclusion that the x-component must relate to the frictional force in some way, so;

$$\boldsymbol{f_s = \mu_s Mgcos\theta_s} \qquad \text{Equation 3}$$

Where we have included the coefficient of static friction – the very thing we are looking to find! If we now take equations 2 and 3 and substitute them in to equation 1 we can create an expression involving things we can measure to find the coefficient of static friction!

$$\boldsymbol{Mgsin\theta_s = \mu_s Mgcos\theta_s} \qquad \text{Equation 4}$$

Solving this equation for **μ_s** is simple and yields a simple relation involving the tangent of the angle. Notice that the mass is not important! What does this mean for the coefficient of static friction?

<u>Kinetic Friction:</u>

In a similar set-up to that of the static friction experiment, the coefficient for kinetic friction can also be found. If the block is tapped lightly as the angle θ is increased, the block will eventually slide down the plane with a constant velocity. Thus we can write the following:

$$\boldsymbol{F_d = f_k} \qquad \text{Equation 5}$$

Notice how this equation looks strikingly similar to equation 1 from earlier? This is a property we like to call symmetry. When you have symmetry, you can cut out a lot of extra steps – making things much simpler. Since equation 5 looks like equation 1, we can correctly assume that the solution for the coefficient of kinetic friction will look similar to that for the coefficient of static friction. So, using the same reasoning as above, we can conclude that

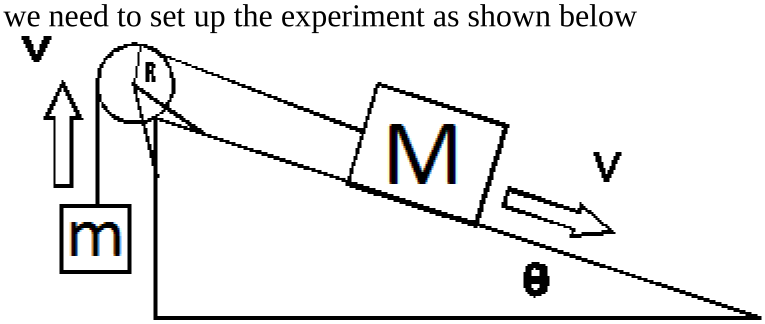
$$\boldsymbol{\mu_k = \tan\theta_k} \qquad \text{Equation 6}$$

Which allows us to calculate the coefficient of kinetic friction between the block and the inclined plane.

<u>Part II: Kinetic Friction</u>

In this part of the experiment, only the kinetic coefficient of friction will be found, and we will do it by some other means than before.

To begin, we need to set up the experiment as shown below



Now, place the **minimum mass, m1**, on the mass hanger such that the block moves up the incline with constant speed. When this occurs:

$$\boldsymbol{m_1 g = F_d + f_k} \qquad \text{Equation 7}$$

Now, place the **maximum mass, m2**, on the mass hanger that will allow the block to move down the incline with constant speed. When this occurs:

$$\boldsymbol{F_d = m_2 g + f_k} \qquad \text{Equation 8}$$

With these two equations, we can come up with a single expression for the coefficient of kinetic friction by simply substituting equation 8 into equation 7. Show your work for this on your calculations page. Now that we have two values for the coefficient of kinetic friction, we can compare them. Use the percent difference calculation to compare the values of the coefficient of kinetic friction that you got from Part I and Part II.

Conclusion:

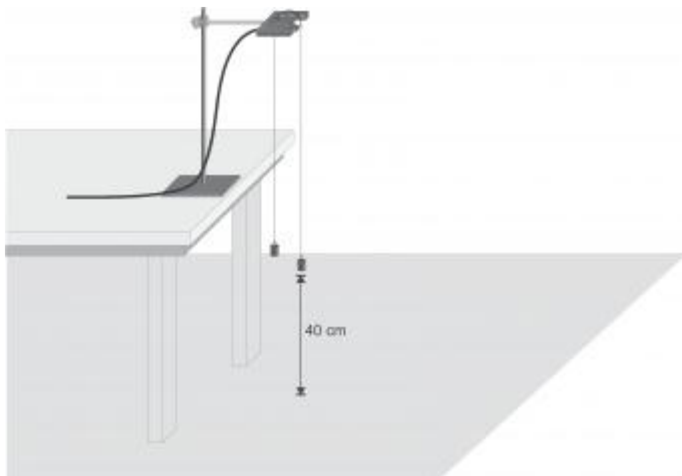
Make sure to draw free-body diagrams for all parts of the lab, as well as show all calculations used to find the coefficients of friction. Pay close attention to the differences in Part I and Part II – how are the expressions used to find the coefficient of kinetic friction similar? How are they different? Is one way better than the other? Why or why not? Calculate a percent difference between the two.

University Physics Workbook Part I
Atwood's Machine (Explore F = ma)

Atwood's Machine

A classic experiment in physics is the *Atwood's Machine*: Two masses on either side of a pulley connected by a light string. When released, the heavier mass will accelerate downward while the lighter one accelerates upward at the same rate. The acceleration depends on the difference in the two masses as well as the total mass.

In this lab, you will determine the relationship between the two factors which influence the acceleration of an Atwood's machine using a Photogate for measuring the acceleration.



Purpose:

Use a Photogate to study the acceleration of an Atwood's machine.

Determine the relationships between the masses on an Atwood's machine and the acceleration.

Materials:

Computer	Vernier Photogate with pulley
Vernier computer interface	Mass set
Logger *Pro*	string

Some Preliminary Questions:

1) If two equal masses are suspended from either end of a string passing over a pulley, what kind of motion do you expect to occur? Why?
2) For an Atwood's machine, how would you expect the acceleration to change if you:
3) Move mass from one side to the other, keeping the total mass constant?
4) Gradually increase the mass of both sides?
5) Why do the two masses have the same acceleration?
6) Draw a free-body diagram of the left side mass. Draw another FBD of the right side mass. Include all forces acting on each mass.

Procedure:

<u>Part I Constant Total Mass</u>

1) For this part of the experiment, you will keep the total mass used constant, but move masses from one side to the other. The difference in masses changes.

2) Set up the Atwood's machine. Be sure the heavier mass can move at least 40 cm before hitting the floor (or table top).
3) Connect the Photogate with the pulley to DIG/SONIC 1 of the interface.
4) Open the file indicated on the whiteboard at the front of the class. A graph of velocity vs. time will be displayed.
5) Arrange a collection of masses totaling 200g on m_2 and a 200g mass on m_1. What is the acceleration of this combination? Record your values for mass and acceleration in a data table.
6) Move 5g from m_2 to m_1. Record the new masses in the data table.
7) Position m_1 as high up as it can go. Click the collect button to begin data collection. Steady the masses so they are not swinging. Wait one second and release the masses. Catch the masses before they hit the floor or the other mass strikes the pulley.
8) Click the examine button and select the region of the graph where the velocity was increasing at a steady rate. Click the linear fit button to fit the line $y=mx+b$ to the data. Record the slope, which is the acceleration, in the data table.
9) Continue to move the masses from m_2 to m_1 in 5g increments, changing the difference between the masses, but keeping the total constant. Repeat steps 6-7 for each mass combination. Repeat this step until you get at least five different combinations.

<u>Part II Constant Mass Difference</u>

10) For this part of the experiment, you will keep the difference in mass between the two sides of the Atwood's machine constant and increase the total mass.
11) Put 120g on m_1 and 100g on m_2.
12) Repeat steps 6-7 to collect data and determine the acceleration.
13) Add mass in 20g increments to both sides, keeping a constant difference of 20 grams. Record the resulting mass for each combination in a data table. Repeat the procedure for each combination of mass until you have at least five different combinations.

Data Table

An example of what your data table should look like is given below.

Part I: Total Mass Constant

Trial	m_1(g)	m_2(g)	Acceleration (m/s2)	Δm (g)	m_T (g)
1					
…					

Analysis

For each trial, calculate the difference between *m1* and *m2*. Enter the result in a column labeled Δm.

For each trial, calculate the total mass in grams. Enter the result in a column labeled m_T.

Disconnect all sensors and choose New from the File menu. Plot a graph of acceleration vs. Δm, using the Part I data. Based on your analysis of the graph, what is the relationship between the mass difference and the acceleration of an Atwood's machine?

Plot a graph of acceleration vs. total mass, using the Part II data. Based on your analysis of the graph, what is the relationship between total mass and the acceleration of an Atwood's machine?

Develop a single expression for the acceleration of an Atwood's machine, combining the results of the previous two steps in the analysis.

Extensions for Developing Your Conclusion

Draw another free-body diagram for some mass *m1* and *m2*. Using these diagrams, apply Newton's second law to each mass. Assume that the tension is the same on each mass and that they have the same acceleration. From these two equations, find an expression for the acceleration of *m1* in terms of *m1*, *m2*, and *g*. Compare the expression to your result in step 5 of the analysis.

For each of the experimental runs you made, calculate the expected acceleration using the expression you found with Newton's second law of motion and the specific masses used. Compare these figures with your experimental results using either a percent difference or percent error calculation. Are the experimental acceleration values low or high? Why?

Some other questions to consider when writing your conclusion: Could an unknown mass be determined using an Atwood's machine? How does the force exerted upward by the pulley change as the system begins accelerating? Why? How does the tension in the string change as the masses start to move? Or does it?

University Physics Workbook Part I
Collisions (1D Energy & Momentum Conservation)

The collision of two carts on a track can be described in terms of momentum conservation and, in some cases, energy conservation. If there is no net external force experienced by the system of two carts, then we expect the total momentum of the system to be conserved. This is true regardless of the force acting between the carts. In contrast, energy is only conserved when certain types of forces are exerted between the carts.

Collisions are classified as elastic (kinetic energy is conserved), inelastic (kinetic energy is lost) or completely inelastic (the objects stick together after collision). Sometimes collisions are described as super-elastic, if kinetic energy is gained. In this experiment you can observe most of these types of collisions and test for the conservation of momentum and energy in each case.

Purpose:

- Observe collisions between two carts, testing for the conservation of momentum.
- Measure energy changes during different types of collisions.
- Classify collisions as elastic, inelastic, or completely inelastic.

Materials:

Computers
Vernier computer interface
Logger Pro
Two Vernier Motion Detectors
Dynamics cart track
Dynamics carts (2)

<u>Procedure</u>

1) Measure the masses of your carts and record them in your data table. Label the carts as cart 1 and cart 2.
2) Set up the track so that it is horizontal. Test this by releasing a cart on the track from rest. The cart should not move.
3) Practice creating gentle collisions by placing cart 2 at rest in the middle of the track, and release cart 1 so it rolls toward the first cart, magnetic bumper toward magnetic bumper. The carts should smoothly repel one another without physically touching.
4) Place a Motion Detector at each end of the track, allowing for the 0.15 m minimum distance between detector and cart. Connect the Motion Detectors to the DIG/SONIC 1 and DIG/SONIC 2 channels of the interface. If the Motion Detectors have switches, set them to Track.
5) (Optional) Open the file “18 Momentum Energy Coll” from the Physics with Vernier folder.
6) Click “collect” to begin taking data. Repeat the collision you practiced above and use the position graphs to verify that the Motion Detectors can track each cart properly throughout the entire range of motion. You may need to adjust the position of one or both of the Motion Detectors.
7) Place the two carts at rest in the middle of the track, with their Velcro bumpers toward one another and in contact. Keep your hands clear of the carts and click “Zero”. Select both sensors and click “ok”. This procedure will establish the same coordinate system for both Motion Detectors. Verify that the zeroing was successful by clicking “collect” and allowing the still-linked carts to roll slowly

across the track. The graphs for each Motion Detector should be nearly the same. If not, repeat the zeroing process.

Part I: Magnetic Bumpers

8) Reposition the carts so the magnetic bumpers are facing one another. Click "collect" to begin taking data and repeat the collision you practiced in Step 3. Make sure you keep your hands out of the way of the Motion Detectors after you push the cart.
9) From the velocity graphs you can determine an average velocity before and after the collision for each cart. V_{10} = original velocity of cart 1, V_{20} = original velocity of cart 2, V_{1f} = final velocity of cart 1, and V_{2f} = final velocity of cart 2. To measure the average velocity during a time interval, drag the cursor across the interval. Click the Statistics button to read the average value. Measure the average velocity for each cart, before and after collision, and enter the four values in the data table. Delete the statistics box.
10) Repeat Step 9 as often as required to obtain reliable data. How much is that? Record the velocities in your data table.

Part II: Velcro Bumpers

11) Change the collision by turning the carts so the Velcro bumpers face one another. The carts should stick together after collision. Practice making the new collision, again starting with cart 2 at rest.
12) Click "collect" to begin taking data and repeat the new collision. Using the procedure in Step 9, measure and record the cart velocities in your data table.
13) Repeat the previous step as a second run with the Velcro bumpers.

Part III: Velcro to Magnetic Bumpers

14) Face the Velcro bumper on one cart to the magnetic bumper on the other. The carts will not stick, but they will not smoothly bounce apart either. Practice this collision, again starting with cart 2 at rest.
15) Click "collect" to begin data collection and repeat the new collision. Using the procedure in Step 9, measure and record the cart velocities in your data table.
16) Repeat the previous step as a second run with the Velcro to magnetic bumpers.

Data Tables

Mass of cart 1 (kg)	Mass of cart 2 (kg)

Run	V_{1o} (m/s)	V_{2o} (m/s)	V_{1f} (m/s)	V_{2f} (m/s)
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

Run	P_{1o} (kg m/s)	P_{2o} (kg m/s)	P_{1f} (kg m/s)	P_{2f} (kg m/s)	P_{To} (kg m/s)	P_{Tf} (kg m/s)	*Err*
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.

Run	E_{1o} (J)	E_{2o} (J)	E_{1f} (J)	E_{2f} (J)	E_{To} (J)	E_{Tf} (J)	*Err*
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.

ANALYSIS

1) Determine the momentum ($P = mv$) of each cart before the collision, after the collision, and the total momentum before and after the collision (see Step 9 above for meaning of subscripts). Calculate the error of the total momentum comparing before and after the collision. What is the sensible error statistic to use? Enter the values in your data table.
2) Determine the kinetic energy ($E = ½\ mv^2$) for each cart before and after the collision. Calculate the error of the total energy comparing before and after the collision. Enter the values in your data table.
3) If the total momentum for a system is the same before and after the collision, we say that momentum is conserved. If momentum were conserved, what would be the error in total momentum?
4) If the total kinetic energy for a system is the same before and after the collision, we say that kinetic energy is conserved. If kinetic energy were conserved, what would be the error in total kinetic energy?
5) Inspect the momentum error. Even if momentum is conserved for a given collision, the measured values may not be exactly the same before and after due to measurement uncertainty. The error should be small. Is momentum conserved in your collisions?
6) Repeat the preceding question for the case of kinetic energy. Is kinetic energy conserved in the magnetic bumper collisions? How about the Velcro collisions? Is kinetic energy consumed in the third type of collision studies? Classify the three collision types as elastic, inelastic, or completely inelastic.

University Physics Workbook Part I
Energy Conservation with a Spring

When a spring is stretched a distance *d* from its equilibrium position, work has been done on the spring. That work is stored in the spring as elastic potential energy, given by the following equation:

$$PE_{Spring} = \frac{1}{2}kd^2$$

The constant *k* is a property of the spring. It is a measure of how much force is required to stretch the spring. For these springs, the average value of *k* is about 9.5 N/m.

When the spring is hanging vertically, as in the setup for our lab today, the spring also has gravitational potential energy given by the following equation:

$$PE_{Gravity} = mgh$$

The height *h* will be measured as a distance above the tabletop.

When the spring in our experiment is stretched and released, it undergoes simple harmonic motion. At any point in the oscillation, if we sum all of the energy of the spring, it is a constant value.

$$PE_{Spring} + PE_{Gravity} + KE = Constant$$

But, at the top of its oscillation, it stops momentarily before it falls back down. At the bottom of its oscillation, it stops momentarily before it is pulled back up. Thus the kinetic energy at each of these positions is zero. The potential energies still exist at these points.

Purpose:

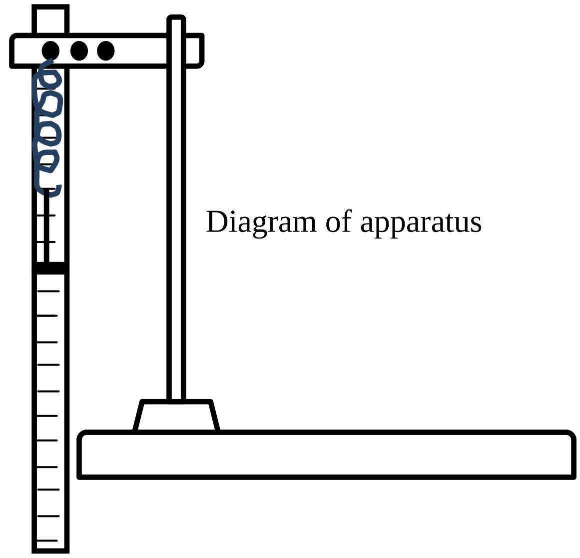
To experimentally test the principle of conservation of energy where gravitational and spring forcers are involved.

Materials:

Ring Stand
Spring
Mass Hanger
Pendulum Clamp
Meter Stick
Masses

Procedure:

1) Our goal today is to calculate the potential energies at the top of the oscillation and the bottom of the oscillation. We will compare them in an effort to show that the total energy is conserved.
2) Set up the apparatus as demonstrated by the instructor. Make sure the 0cm mark of your meter stick is on the table. In your data table, record the position of the upstretched spring as h_o.



3) Place a 200g mass on the hook on the spring. Choose a position 2-3 cm below position h_0. This will be your "drop height" – record it in your data table as h_1. The distance (h_0 - h_1) is the distance d_1 that the spring has stretched. Record d_1 in your data table also.
4) Calculate the elastic potential energy and the gravitational potential energy at the top of the oscillation and record the values in your data table.
5) Set up the Vernier ultrasonic detector under the 200 g mass.
6) Start Vernier data collection and then release the mass to begin the simple harmonic motion. By careful observation and repeated trials, you can estimate the lowest point to which the mass descends before returning upward. This position will be recorded in your data table as h_2. The distance ($h_0 – h_2$) is the distance d_2 that the spring has stretched at its lowest point. How close does your observation come to the Vernier reading?
7) Note: To measure the lowest position, you need to make your measurement on the first 2-3 oscillations. After that, the motion begins to damp out and your measurements will not be as accurate.
8) Calculate the elastic potential energy and the gravitational potential energy at the bottom of the oscillation and record the values in your data table.
9) Find the total energy at the top of the oscillation and at the bottom of the oscillation and record the totals in your data table.
10) Find the percent difference in the two total values.

Data Table:

Unstretched spring height h_0		
Drop height h_1		
Distance spring stretched at the top $d_1 = h_0 - h_1$		
Energy at Top:	PEspring	PEgravity
Lowest height h_2		
Distance spring stretched at the bottom $d_2 = h_0 – h_2$		
Energy at Bottom:	PEspring	PEgravity

Total energy at top =	Total energy at bottom =

Percent difference: ________________________ $\%difference = \left|\frac{E_t - E_b}{\frac{E_t + E_b}{2}}\right| x\, 100$

Questions for Conclusion:

1) Assume that the total energy at the top of the oscillation is the total energy in the system, find the gravitational potential energy, the elastic potential energy, and the kinetic energy at the midpoint of the oscillation.
2) What are some possible reasons for any error you experienced? Remember, human error is <u>not</u> a valid source of error!
3) Was the purpose of this lab accomplished (if it was, how did you demonstrate that energy is conserved in this system? If not, explain why not).

University Physics Workbook Part I
2D Collisions (Momentum Conservation)

When objects collide, energy and momentum are both transferred between the colliding objects. If certain properties of one of the colliding objects are known, then properties of the other object can also be known.

Purpose:

To experimentally test the principle of conservation of momentum in a 2 dimensional collision.

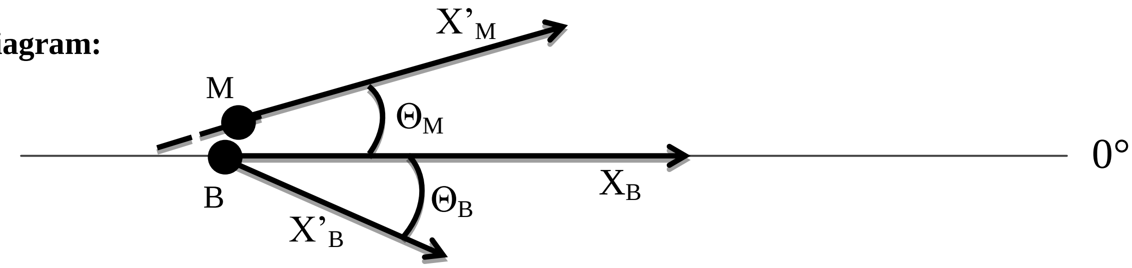
Materials:

Marble Launcher
Marbles
Target Paper with Carbon Paper
Triple Beam Balance
Meter Stick
Plumb-Bob

Procedure:

1) Specific procedures for this experiment are to be established by each working group of students. What follows is a general statement of procedures and practices.
2) The apparatus consists of collision balls and a curved, grooved track. The grooved track is to be mounted on the edge of the table such that upon rolling down the track the steel ball will leave horizontally and fall freely under the influence of gravity until hitting the floor. By taping a large sheet of paper on the floor, the impact points can be marked by allowing the steel ball to strike carbon paper placed at the point of impact. Place a piece of paper over the carbon paper to prevent tearing. The point at which the ball leaves the track can be projected on the paper by means of a plumb-bob.
3) By allowing the steel ball to roll from the same position each time, it should have the same horizontal velocity each time. To determine this velocity, run 6-8 trials and determine the average point of impact and percent difference. This gives the measurable quantities of height fallen and distance traveled horizontally during the fall. Use the equations motion with constant acceleration to determine the time of flight and the horizontal velocity of the ball at the start of the fall.
4) By positioning a second ball on the support provided, a collision can be made to occur when the first call rolls down the track under the initial conditions. Care should be exercised to position the second ball so that the collision occurs just as the first ball leaves the track. Again, 6-8 trials should be run so that the average impact points of the two balls can be found.
5) Since the mass of each ball is measurable, the momentum of each ball before and after the collision can be calculated. Find the horizontal momentum of the steel ball-marble system before and after the collision and compare them by finding the percent difference between the two values. Interpret your results. You can also investigate conservation of kinetic energy. Is this collision elastic or inelastic? What are the errors?

Diagram:



Data Table:

$Y=$	$v_{B=}$	$P_{B=}$
$m_B=$	$v_B'=$	$P_B'=$
$m_M=$	$v_M'=$	$P_M'=$
$x_B=$	$x_B'=$	
$\theta_B=$	$\theta_M=$	

University Physics Workbook Part I
Circular Motion (Centripetal Force)

Purpose

To investigate centripetal force relationships for an object in uniform circular motion.

Procedure

If a mass is forced to travel in a circular path, it must be continually accelerated even if the speed is constant. This is because the direction of its velocity is continually changing. This acceleration is along the radius connecting the body with the center of its circular path. For a path of radius R at linear speed v, the "center-seeking" or centripetal acceleration a is given by

$$a = \frac{v^2}{R}$$

Newton's second law gives us the magnitude of the centripetal force **F** necessary for this centripetal acceleration on a body of mass m.

$$F = ma = \frac{mv^2}{R}$$

Figure 1 is a diagram of a system in which a mass (a stopper, labeled mass s) is being whirled at radius R about a glass tube which serves as a bearing for the string. As the mass whirls, a second mass (labeled mass w) attached to the opposite end of the same string provides the necessary centripetal force. If the speed of the rotation is too small, mass w begins to move downward. If the speed of rotation is too great, mass w begins to move upward.

Assemble an apparatus similar to the one in Figure 1. Start by making mass w on the order of 20 g and total string length about 1.5 m. Thread the string through the glass tube and attach mass w to the string. Tie mass s securely to the other end of the string. Move to an area where there is plenty of room and careful1y whirl mass s above your head. You will find that mass w moves up and down as the speed of rotation is changed. With a little practice, you can find a combination of speed of rotation, mass w, and radius of rotation which will keep mass w in stationary equilibrium.

Once you have found such a combination, have your lab partner time 20 revolutions of mass s. Before you stop rotating mass s, grasp the string at the bottom of the tube and hold it there as you stop. Continue holding the string and mark it at the top of the glass tube with a pen. The distance from the pen mark to the center of mass s is the radius of rotation R.

Divide the total time to make 20 revolutions by the number of revolutions (20). This is the Period (T)

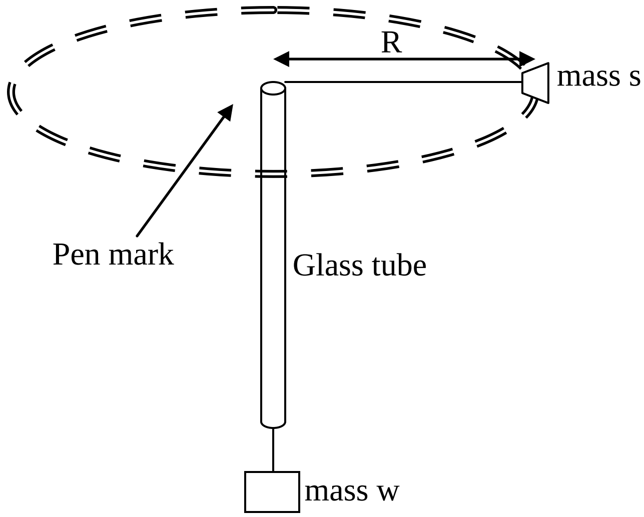
$$T = \frac{\text{total time for 20 revolutions}}{\text{20 revolutions}}$$

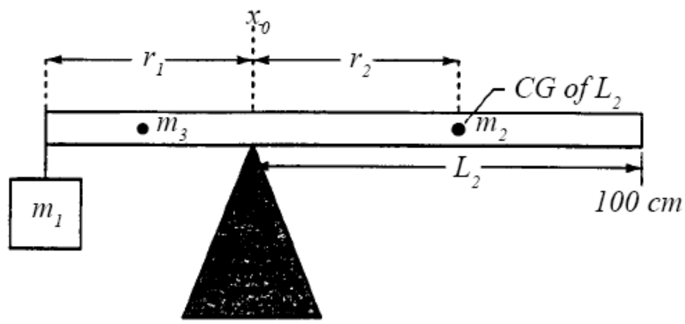
Each revolution is $2\pi R$ in length; therefore, the linear speed of mass s is given by

$$v = \frac{2\pi R}{T}$$

Determine the required force to make the stopper stay in its orbit and compare that force with the tension provided by the weight. Perform 6 trials using at least 2 stoppers, 2 weights, and various radii. Create and label a data table with the following quantities and their units: #Trials, Ms, Mw, Time for 20 Rev. with subdivisions of a, b, & avg, radii, T, *v*, Fc, mg, %difference.

III. Diagram





Purpose

In this experiment, you will investigate torques on rigid bodies and static equilibrium.
Equipment

- 1 Lab Balance
- 1 Meter stick
- 1 Balance Stand for meter stick
- 1 Set of Mass Hangers for meter stick
- 1 Hooked Mass Set
- 1 Small, Unknown Metal Mass

Introduction

Consider, for example, an ordinary chair. It appears to be a rigid body. Is it in equilibrium as it sits on the floor? If, as often happens, one of its legs is a bit short, it will not be in equilibrium; instead, it will wobble about the two diagonally-opposite longer legs. But if an elephant sits on the chair, it will most definitely be in equilibrium with all four legs resting solidly on the floor.

What has changed? Only the chair's shape. The elephant has distorted it. It is not a rigid body under these conditions.

In this experiment, you will investigate truly rigid bodies and static equilibrium. The static equilibrium condition is very important in civil engineering, applying to bridges, dams, buildings, statues, and balconies, and in our daily lives, applying to our ability to stand up, to drive around corners without overturning, and to slide a stein of beer the length of the bar without spilling it.

Analyzing static equilibrium conditions is an essential part of architectural engineering. The designer needs to identify all the forces and torques that act on a structural element, and to ensure through design and materials selection that the element will safely tolerate the loads to be exerted on it.

Two conditions must be met for a rigid object subject to a combination of external forces to be in mechanical equilibrium:

The vector sum of all the external forces acting on it must be zero. Translational equilibrium:

$$\sum_i \vec{F} = 0$$

The vector sum of all the *individual* torques about an arbitrary axis must be zero. Rotational equilibrium:

$$\sum_i \vec{\tau} = 0$$

To be in static equilibrium, a rigid object must also be in rotational equilibrium.

The concept of center of mass is what allows us to study the motion and equilibrium of extended (real world) objects as if they were point objects. By considering the translational motion of an object's center of mass (the motion of a point mass), and the rotational motion of the object about its center of mass, we can determine the complex motion of any extended object.

In this experiment, you will examine torques, rotational equilibrium, and center of mass as they apply to a rigid object. The rigid object will be an ordinary wooden meter stick. By measuring the forces and calculating the torques acting on this meter stick in different situations, you will experimentally verify the two equilibrium equations. In doing this, you will learn to

- Describe mechanical equilibrium of a rigid object
- Explain the center of mass concept
- Explain how a laboratory balance measures mass

Theory

Equilibrium

A rigid body in static equilibrium must necessarily be in rotational equilibrium. Torque about some axis of rotation (also called *moment of force*) results from a force being exerted at a point ***not on the axis***. Torque is defined as the vector product of the force and the displacement to the axis:

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Therefore, the magnitude of the torque is

$$\tau = rF\sin(\theta)$$

The measurement unit for torque is the newton-meter (N m), which you should not confuse with the unit of work (1 newton-meter = 1 joule).

Torque is a vector quantity. Its direction is normal to the plane containing ***r*** and ***F***. When you cross ***r*** into ***F*** using the right-hand rule, your thumb points in the direction of the radius (outward sense – radius vector tail is at center and tip is on circle), fingers in the direction of the force, and palm points in direction of torque. For convenience, torques are often designated by the circular directions of motion that they tend to cause (clockwise or CW, and counter-clockwise or CCW).

A rigid body can rotate about a specific axis in only two directions, CW or CCW. CW torques produce CW rotational motion and CCW torques cause CCW rotational motion. Rotation will not begin or change if the applied torques are balanced (if the system is in rotational equilibrium). The condition for rotational equilibrium is

$$\sum_i \vec{\tau} = \sum_i \tau_{CCW} + \sum_i \tau_{CW} = 0$$

where τ_{CW} is CW torque and τ_{CCW} is CCW torque. Conventionally, the CCW direction is positive, and the CW direction is negative. When you put your right hand on a clock with your thumb pointing perpendicular to the face, the fingers point in the positive, CCW, direction. Digressing, CW is the direction a shadow of the sundial moves during the day in the Northern Hemisphere and as clocks were developed that convention was maintained.

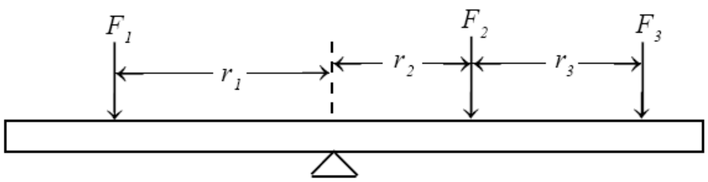


Figure 8.1. Torques applied to bar

For the system in Figure 8.1, the condition for rotational equilibrium becomes:

$$F_1 r_1 = F_2 r_2 + F_3 r_3$$

In this case the counterclockwise torque is $F_1 r_1$ and the clockwise torques are $F_2 r_2$ and $F_3 r_3$. Note that the forces F_i may also be expressed as weights according to Newton's Second Law ($W_i = F_i = m_i g$). In this equation, the weight W has units of N, the mass m is in kg, and the gravitational acceleration g is in m/s^2.

You can use the equation of balanced torques to find an unknown quantity such as a moment arm. Assume the bar above is in static equilibrium and the forces exerted on the bar are due to three masses hanging at the indicated positions. Then, dividing the previous equation by g, equilibrium is given by:

$$m_1 r_1 = m_2 r_2 + m_3 r_3$$

or

$$r_1 = \frac{m_2 r_2 + m_3 r_3}{m_1}$$

If $m_2 = m_3 = 100$ gm, $m_1 = 200$ gm, $r_2 = 40$ cm, and $r_3 = 60$ cm, r_1 must be

$$r_1 = \frac{(100 \text{ gm})(40 \text{ cm}) + (100 \text{ gm})(60 \text{ cm})}{(200 \text{ gm})} = 50 \text{ cm}$$

Center of Gravity

The *center of gravity* of an object is the point where all of the weight of that object (*mg*) may be concentrated for the purpose of determining the torque gravity exerts on it. The weights of the infinitesimal mass particles making up a rigid object create torques about the object's center of gravity. A wooden meter stick of uniform cross section, for example, may be considered as made up of many point masses that are in rotational equilibrium about its center of gravity at the 50 cm point. The ruler can be balanced (supported in rotational equilibrium) on a fulcrum located at its center of gravity.

The *center of mass* of this same ruler would be at the same location as its center of gravity as long as the acceleration due to gravity g is uniform. For a symmetrical stick with a uniform mass distribution, its center of mass and its center of gravity will both be located at its center of symmetry.

The concept of linear mass density u is closely related to the concept of uniform mass distribution. The linear mass density of a rigid object is defined as its mass per unit length:

$$\mu = \frac{m}{L}$$

If a meter stick has a mass of 150 gm, its linear mass density becomes (150 gm)/(100 cm) = 1.50 gm/cm = 0.15 kg/m. Assuming a uniform mass distribution for the ruler, every centimeter of it has a mass of 1.50 g, and 60 cm would have 90 g of mass. Keep in mind, however, that laboratory 1-m rulers do not necessarily have uniform mass distribution, especially older ones that are worn on their edges, and such an assumption gives only approximate values.

Procedure

You will use minimal equipment and ***no computer sensors*** in this experiment. In hanging masses from the meter stick, you may suspend them from sliding clamps with hangers or from loops of string. Suspending the hooked masses from loops of string (or rubber bands) is easier since you can consider the strings to be massless. If you use the clamps with hangers, be sure that you add their masses to the hanging mass.

A. Rigid Rod Supported at its Center of Gravity

Measure the mass of the meter stick and record its value above Data Table 8.1.

Case 1. Two masses of known value

1) Supporting the meter stick at its center of mass (point where the meter stick is balanced by itself), hang a 100 gm mass at the 20 cm point. Record this mass m_1 and its moment arm r_1 in your data tables.
2) Balance the ruler by placing a 200 gm mass at an appropriate location on the other side of the fulcrum. Record this mass m_2 and its moment arm r_2 in your data tables
3) Calculate and record the CW and CCW torques and the percent difference between them.

Case 2. Three masses of known value

4) With the ruler still supported at its center of mass, hang a 100 gm mass m_1 at the 15 cm mark and a 200 gm mass m_2 at the 75 cm mark. Hang a 50 gm mass m_3 at the appropriate position so the ruler is in static equilibrium. Record all masses and moment arms in your data tables
5) Compute and record the torques and percent differences as you did in step 4.

B. Rigid Rod Supported at Different Points

6) In Part A, you did not take the mass of the ruler itself into account because the support was located at its center of gravity, and the torque due to its weight was zero.

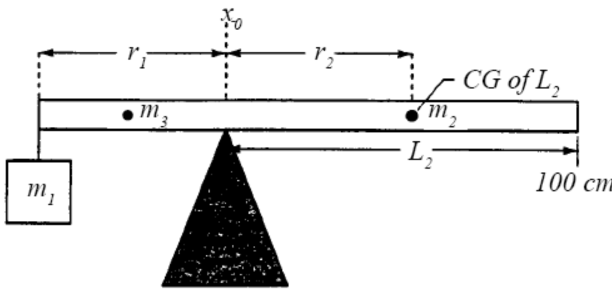


Figure 8.2. Unbalanced rod

7) In this part, the ruler will not be supported at its center of gravity but at other pivot points indicated in general in Figure 8.2. For these cases, you will have to consider the weight of the ruler and compute appropriate torques for each case due to m_2 and m_3.
8) Hang a 100 gm mass m_1 near the zero end of the ruler and slide the support to the appropriate position to achieve static equilibrium. Record the mass and moment arm in your data table.
9) Assuming uniform mass distribution of the ruler, calculate and record the mass m_2 of L_2.
10) Calculate and record the moment arm r_2.
11) For the same setup, calculate and record the moment arm r_3 due to the mass m_3 of that portion of the ruler on the other side of the support.
12) Calculate and record the torques and percent differences as you did in Part A.

C. Center of Gravity

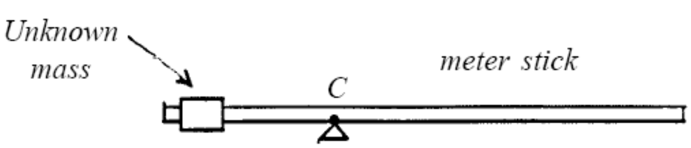


Figure 8.3. Loaded ruler balanced at C

13) Tape an unknown mass near one end of the ruler as shown in Figure 8.3. Keep the mass at the same position for the rest of this part.
14) Measure the mass m_r of the loaded ruler and record it under Data Table 8.3.
15) The center of gravity of the loaded ruler is now at point *C*. Balance the loaded ruler and record the exact position of point *C* in Table 8.3.
16) Hang a mass m_1 of arbitrary value from point *A* near the other end of the ruler. Adjust the support point of the loaded ruler from *C* to *B* until the ruler is again in

static equilibrium (see Figure 8.4). Record the value of m_1 and the moment arms L and x in your data (the weight W of the loaded ruler is considered as acting at point C a distance x from the pivot).

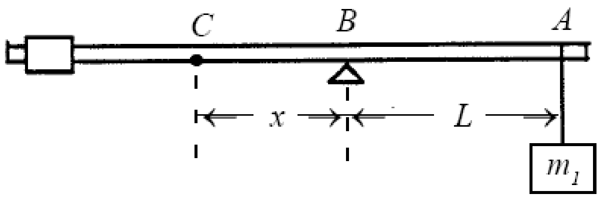
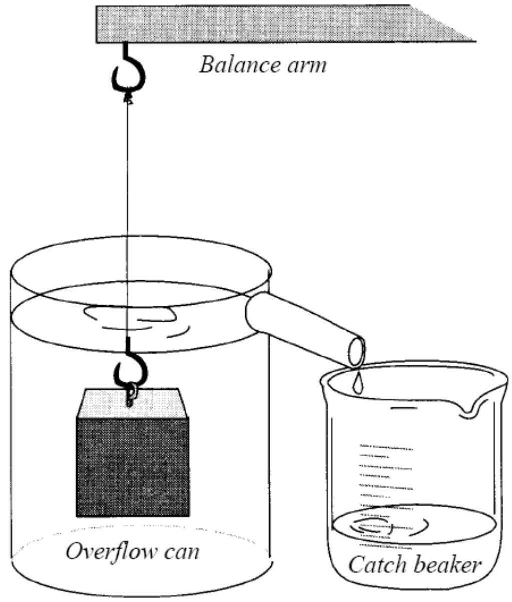


Figure 8.4. Loaded ruler balanced at B

17) Calculate and record the torques and percent differences as you did in Part A.
18) Disassemble the equipment and return it to where you got it from. Clean up around your lab table area.

University Physics Workbook Part I
Archimedes' Principle (Fluids)



Purpose

You will use Archimedes' principle in this experiment to determine the densities of several solids and liquids.

Equipment

- Spring scales calibrated in N (preferred), force sensor, or mass balance rigged so object can be tied to it and lowered into water. There is a procedure to place the mass balance on the table, place a beaker of fluid on it, raise or lower objects into the fluid, and measure forces, however it requires a major revision of this theory and procedure and is therefore outside the scope of this laboratory.
- Lab jack, scissor jack, blocks, pieces of 2x4, or other blocks to raise and lower beakers, etc.
- Overflow can
- If mass balance is used, be sure to convert mass reading to force readings.
- 500-mL beaker
- 1000-mL beaker
- Large ringstand, stand with 90° clamps and crossbar, or other arrangement to hang objects being measured.
- Metal cube with hook. If it doesn't have a hook, tie it with string like a Christmas package.
- Lab balance
- String
- Wooden object with hook. If it doesn't have a hook, tie it with string like a Christmas package.
- Hydrometer or hydrometers capable of measuring liquids heavier and lighter than water

- Metal cylinder with hook. If it doesn't have a hook, tie it with string like a Christmas package.
- Bottle of unknown liquid such as saltwater, ethanol/water mixture, antifreeze/water mixture, etc.

Introduction

A solid object may float or sink when placed in a given fluid (i.e., in a gas or a liquid). If the object floats, it is buoyed up by a force equal to its weight. According to the ancient Greek philosopher Archimedes, the buoyant (upward) force exerted on an object that is either wholly or partially submerged in a fluid is equal to the weight of the amount of fluid displaced by the object. The object will sink when its weight exceeds the weight of the displaced fluid.

Because of Earth's gravity, every fluid substance from the atmosphere to the oceans to a tank full of gasoline has an internal pressure that increases with depth. And because gases are compressible, the greater pressure at a given depth squeezes the fluid into greater density at that depth. The density of an ideal gas is proportional to absolute pressure and inversely proportional to absolute temperature. Liquids are much less compressible, however the density of water increases about 0.3% as you descend to the bottom of the ocean.

We live our lives at the bottom of the atmospheric ocean which exerts about 101,300 N of force on every square meter of our bodies (10.13 N/cm^2). We don't notice this pressure, of course, because the fluids inside our bodies exert a balancing outward pressure. This balance is not automatic. Sometimes driving down a mountain the tube leading to our inner ear is temporarily blocked and we feel this pressure until we yawn to open the tube to "pop" our ears. The pressure change may be great enough that we cannot breathe normally and we must surround our bodies with air at normal atmospheric pressure if we ascend into the atmosphere or descend into the ocean.

Because of the vertical pressure gradient in fluids, the pressure on the lower surface of a submerged object is always greater than that on the upper surface. This is the root cause of the buoyant force.

Specific gravity is really nothing but relative density; that is, it is a unit-less ratio between the density of an object or a substance relative to the density of pure water, with both values of density expressed in the same units. Specific gravity is much more useful when dealing with different systems of units. The specific gravity of pure water is always 1.0, however the density is 1.0 gm/cm^3, 1000 kg/m^3, or 62.4 lb/ft^3. Thus the density of steel is 7.8 gm/cm^3, 7800 kg/m^3, or 487 lb/ft^3, but the specific gravity is always 7.8 independent of the system of units.

You will use Archimedes' principle in this experiment to determine the densities of several solids and liquids. When you finish the experiment, you will be able to:

1) Determine whether or not an object will float or sink in a fluid if you know the density of each.
2) Explain the difference between density and specific gravity.
3) Determine the density or specific gravity of a solid or fluid whether it floats or sinks.

Theory

A. *Objects Denser Than Water*

The buoyant force is described by Archimedes' principle as: *an object, when placed in a fluid, is buoyed up by a force equal to the weight of the fluid displaced by the object.* The principle applies to an object either entirely or partially submerged in the fluid. The magnitude of the buoyant force depends *only* on the weight of the displaced fluid, and not on the object's weight. Using Archimedes' principle, you can deduce that an object:

1) Will float in a fluid if the object's density is less than the fluid's density ($\rho_o < \rho_f$).
2) Will sink if the object's density is greater than the fluid's density ($\rho_o > \rho_f$).
3) Will remain in equilibrium at a given submerged depth if the object's density is exactly equal to the fluid's density at that depth ($\rho_o = \rho_f$).

The buoyant force on a floating object F_b is related to the properties of the displaced fluid by:

$$F_b = m_f g = \rho_f V_o g \qquad \textbf{\textit{Equation 1}}$$

where ρ_f is the density of the fluid, V_o is the volume of the submerged part of the object, and g is the acceleration due to gravity.

When working with forces it is important for the student to be able to draw a free body diagram and working with buoyant force is no exception. Figure 1 following shows the free body diagrams (a) in air and (b) in water.

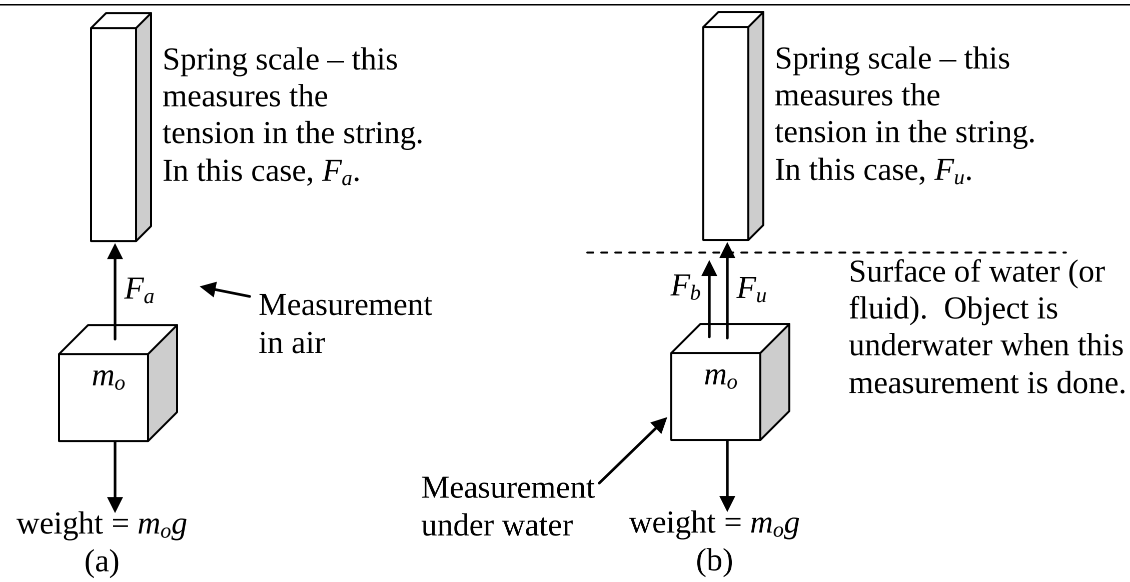


Figure 1 – Free body diagrams of forces on object of mass m_o. Neither the mass nor the weight of the object changes, however on the right hand side depicting the case of measurement underwater (or under some other fluid) there is a buoyant force, F_b. The spring scale (or force sensor, etc.) simply measure the tension in the string attached to the object. On the left hand side, measurement in air, this tension is equal and opposite to the weight (since the object is at rest). Note the density of air is quite small and we therefore presume the buoyant force of air on the object is negligible. On the right hand side, measurement of the object while it is under the surface of a fluid, the spring scale reads F_u – equal and opposite to the difference between the weight and buoyant force.

The volume of the submerged object oriented vertically is equal to its cross-sectional area A multiplied by the height h of the submerged part. Note that the force of the fluid on the top and bottom equals:

$$F_{bottom} = AP_{bottom} \text{ and } F_{top} = AP_{top} \qquad \textbf{\textit{Equation 2}}$$

Remember that the pressure on the bottom is slightly greater than the pressure on the top and the difference equals:

$$P_{bottom} = P_{top} + \rho_f hg$$ ***Equation 3***

The buoyant force is $F_{bottom} - F_{top}$. Using this and rearranging Equations 2 and 3 tells us the buoyant force:

$$F_b = \rho_f Agh$$ ***Equation 4***

Since $Ah = V_o$ it's easy to observe Equation 4 is equivalent to Equation 1. This is a linear relationship between F_b and h, so if you lower the cylinder into a fluid as you measure its weight, then plot F_b vs. h, the slope of the plotted straight line will be $\rho_f Ag$, that is, directly proportional to the density of the fluid. This is a cool way to determine the density of an unknown fluid.

You can determine the density of an unknown solid object in a similar fashion. It's easy to measure the mass of an object, but unless it has a regular shape it's not so easy to measure its volume. But Archimedes showed us how to measure volume by measuring weight.

When the object is completely submerged in a fluid, the measured force (but not its mass) will decrease by an amount equal to the upward buoyant force the water exerts on it (refer to Figure 1). So:

$$\Delta F_o = F_a - F_u = F_b$$ ***Equation 5***

where $F_a = m_o g$ is the weight in air and F_u is the weight when submerged underwater. This upward force is also equal to the weight of the displaced fluid, or:

$$\Delta F_o = W_f = F_b = m_f g = \rho_f g V_f$$ ***Equation 6***

What Equation 6 is telling us is that the buoyant force, F_b, is the weight of the fluid displaced, W_f. If we set up the overflow can as in Figure 2 (as follows), put in water until is overflows (and discard excess water), then drop an object into the overflow can, collect the water that overflows, and weigh the overflow THEN the weight of the overflow will equal the buoyant force.

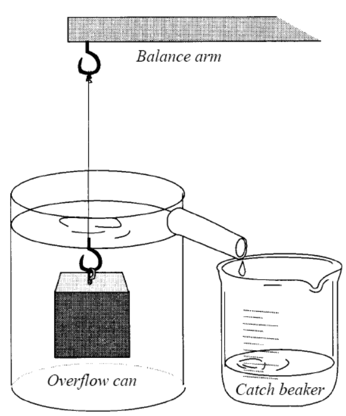


Figure 2 – Experimental arrangement to catch overflow and weigh the overflow water. This weight should equal the buoyant force.

We have a second way to measure F_b per Equations 5 and 6. We weigh the object in air, F_a, and subtract the weight when submerged underwater, F_u. We will compare the two methods. The volume of the water is equal to the volume of the object, so:

$$V_f = V_o = \frac{\Delta F_o}{\rho_f g}$$ ***Equation 7***

The density of the object is therefore (recall F_a is the force reading in air and equals the weight of the object):

$$\rho_o = \frac{m_o}{V_o} = \frac{m_o \rho_f g}{\Delta F_o} = \rho_f \left(\frac{F_a}{F_a - F_u} \right)$$ ***Equation 8***

B. <u>Density of an Unknown Liquid</u>

You can also determine the density of an unknown liquid without measuring the submerged height of the solid object. Recall Equations 4 and 5 and Figure 1. When we measure a sinking object in water we obtain $F_{bw} = \rho_w V_o g = F_a - F_{uw}$ where ρ_w refers to the density of water and when we measure the same object in the unknown fluid we obtain $F_{bf} = \rho_f V_o g = F_a - F_{uf}$. Where the subscript "b" refers to <u>b</u>uoyant force, subscript "w" refers to <u>w</u>ater, subscript "f" refers to unknown <u>f</u>luid, subscript "a" refers to measurements in <u>a</u>ir, and subscript "u" refers to submersion <u>u</u>nder the water or unknown fluid. The ratio of F_{bf} to F_{bw} is given by the following equation:

$$S.G. = \frac{\rho_f}{\rho_w} = \frac{F_{bf}}{F_{bw}} = \frac{F_a - F_{uf}}{F_a - F_{uw}}$$ ***Equation 9***

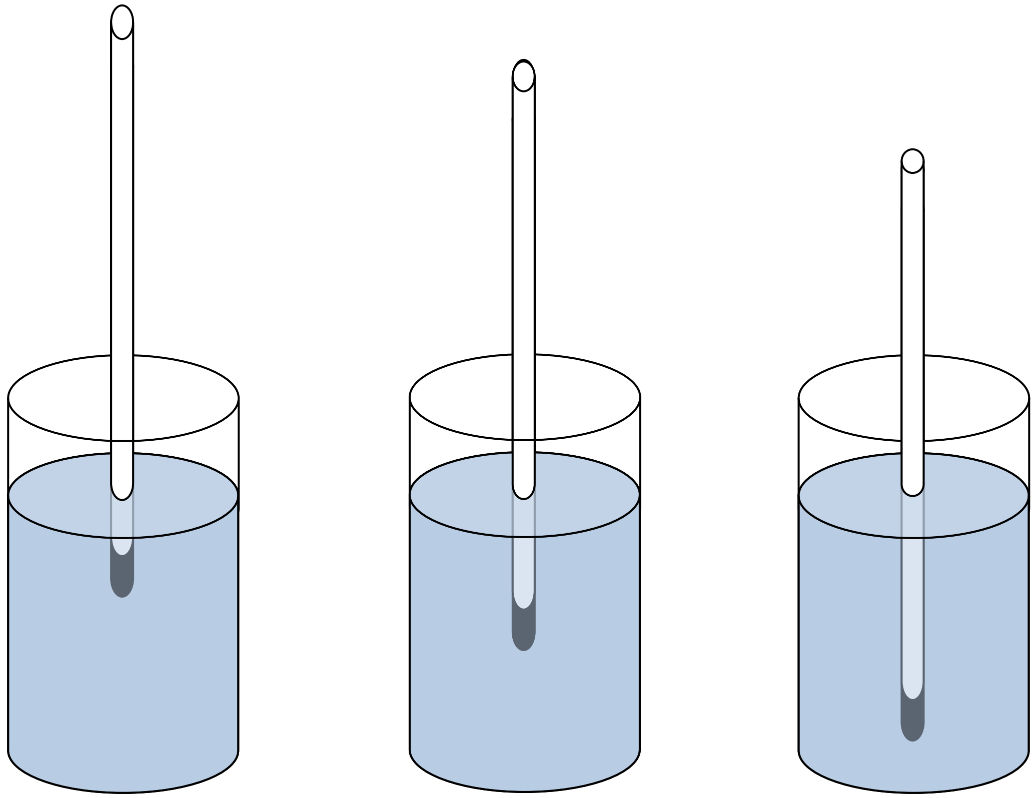
Multiplying the specific gravity of the unknown fluid given by Equation 9 by the density of water gives the density of the unknown fluid.

$$\rho_f = \rho_w (S.G.) = \rho_w \frac{F_{bf}}{F_{bw}} = \rho_w \left(\frac{F_a - F_{uf}}{F_a - F_{uw}} \right)$$ ***Equation 10***

Devices to measure S.G. are known as hydrometers. They are simply closed tubes weighted at the bottom of known mass. The deeper in the fluid the hydrometer goes, more volume of fluid is displaced raising the buoyant force. When the buoyant force equals the weight, the hydrometer stops. For a fluid denser than water the hydrometer rides higher (less deep) than for water. For a fluid less dense than water the hydrometer goes deeper. A scale on the side of the hydrometer is calibrated to indicate specific gravity. This is illustrated in Figure 3.

C. Objects Less Dense Than Water

Most hydrometers are less dense than water and that gives us a hint of how to measure the density of a liquid less dense than water. We will explore two methods – the first being the principle the hydrometer uses. For the most part it's impractical – objects have to be shaped like hydrometers and most are not. Therefore we need a second method – attach a sinker to a floating object to make it sink and measure the forces. In the following section we will develop the theory behind both.



Case (a), $\rho_f > \rho_w$

Hydrometer is shallow displacing less volume. $\rho_f V_h g = F_b = W_h$ where W_h is the weight of the hydrometer (and doesn't change) and V_h is the volume the hydrometer displaces. Therefore if $\rho_f \uparrow$, $V_h \downarrow$ (large fluid density implies small volume displaced)

Case (b), $\rho_f = \rho_w$

Hydrometer in water is at a very specific depth such that $\rho_w V_h g = F_b = W_h$

Case (c), $\rho_f < \rho_w$

Hydrometer is deep displacing more volume. Since $\rho_f V_h g = F_b = W_h$ if $\rho_f \downarrow$, $V_h \uparrow$. Therefore this fluid is less dense than water.

Figure 3 – Hydrometer in three different density liquids. Hydrometer is shallow if fluid is more dense than water, hydrometer is medium if fluid density equals water, and hydrometer is deep if fluid is less dense than water.

The first question is, for an object less dense than water, what fraction of it is above the surface? When an object floats the buoyant force is equal and opposite to the weight. Therefore:

$$F_b = \rho_f V_u g = weight = \rho_o V_o g \qquad \textbf{\textit{Equation 11}}$$

Where V_u is the volume of the object underwater and V_o is the total volume of the object. As previously used, ρ_f is the density of the fluid, ρ_o is the density of the object, and g is the acceleration of gravity. Since $V_o - V_u$ is the volume above water, by rearranging Equation 11 we find the fraction above water is:

$$\frac{V_o - V_u}{V_o} = 1 - \left(\frac{V_u}{V_o}\right) = 1 - \left(\frac{\rho_o}{\rho_u}\right) = \text{fraction above water} \qquad \textbf{\textit{Equation 12}}$$

Typically 60% of a wooden object is above water (however it depends greatly on the type of wood). If this is true, what is the density of this wood? We use Equation 12 to find the answer and it is ρ_o = 0.4 gm/cm^3.

Applying Equation 12 is usually impractical due to the irregular shape of ordinary objects including icebergs. We can, however, tie a sinker to a floating object and measure forces to find the object's density. Let's first look at the free body diagrams in Figure 4 below.

F_a is the spring scale reading in air for the object and sinker, F_s is the spring scale reading when only the sinker is underwater, and F_u is the spring scale reading when both are underwater. The buoyant force acting on the object is simply $F_{bo} = F_s - F_u = \rho_f V_o g$. We can then proceed just like Equations 7 and 8:

$$V_f = V_o = \frac{F_{bo}}{\rho_f g} \qquad \textbf{\textit{Equation 13}}$$

The density of the object is therefore (recall F_o is the weight of the object, force of gravity on the object, and equals $m_o g$):

$$\rho_o = \frac{m_o}{V_o} = \frac{m_o \rho_f g}{F_{bo}} = \rho_f\left(\frac{m_o g}{F_s - F_u}\right) = \rho_f\left(\frac{F_o}{F_s - F_u}\right) \qquad \textbf{\textit{Equation 14}}$$

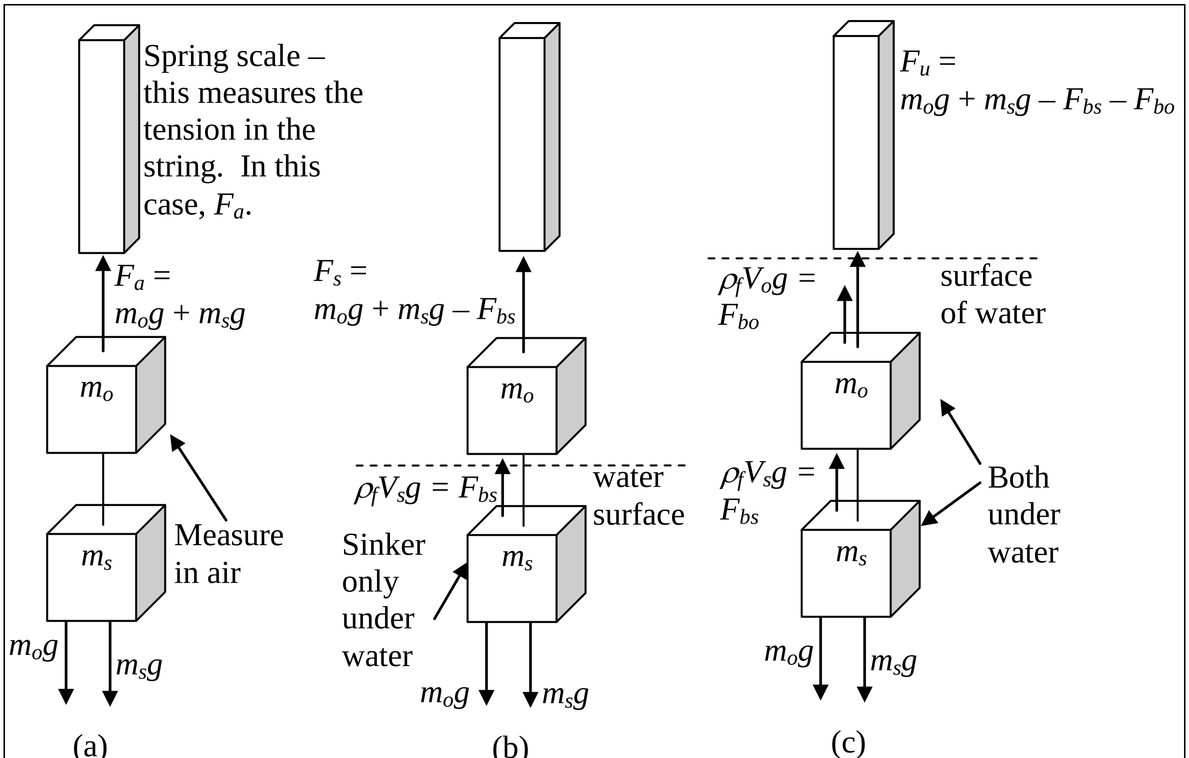


Figure 4 – Free body diagrams of forces on object of mass m_o and sinker of mass m_s. when measured in air, only the sinker is submerged, and when both are submerged.

See! This isn't very hard. Just keep the free body diagrams in mind, figure out forces, and calculate density. Also choose variable that remind you of what they physically mean. You'll notice that the subscript "s" refers to a sunken sinker, "u" refers to both underwater, F_{bo} is the buoyant force on the object, etc. Naming things well is a mnemonic device to help us remember their meaning.

Procedure

You will:

1. Weigh a sinking object in air and under water
2. Weigh the water displaced
3. Weigh a floating object in air and under water (using a sinker)
4. Measure the volume of the floating object, calculate density directly, and compare results with results from using Archimedes' Principle
5. Weigh the sinking object under water and under an unknown liquid
6. Find the specific gravity of an unknown liquid using a hydrometer

From these measurements, you will calculate the densities of the objects and liquids. It is preferable to use spring scales to measure forces directly, however if mass balances are used be sure to convert mass readings to force readings.

A. Density of a Sinking Object

1. Place the spring scale on the ringstand. Alternatively, if using a lab balance, mount the lab balance on a ringstand about two feet high.
2. Using thread tie the object with a loop on the end of the thread.
3. Place the overflow can on a lab jack under the object.
4. Weigh the metal object m_o , calculate its weight, F_a (force reading in air), and record this in Table 1.
5. Weigh the dry catch beaker m_b and record its mass in Table 1.
6. Place the catch beaker beneath the spout of the overflow can as shown in Figure 11.2, and fill the overflow can with water until it runs out the spout. Then empty and replace the beaker.
7. Hang the metal object by the short thread from the hook beneath the balance pan, then raise the lab jack until the object is completely submerged in the water. Weigh the submerged object and record its force, F_u, in Table 1.
8. Remove the sample from the hook under the scale or balance.
9. Measure and record the combined mass of the catch beaker and the overflow water m_{comb}.
10. Record the measured mass m_w of the displaced water.
11. Calculate the weight of the displaced water, W_w, and record this in Table 2.
12. Use Equation 5 to calculate the buoyant force, F_b, and record this in Table 2.
13. Calculate and record the percent difference between W_w and F_b. in Table 2
14. Using Equation 8, calculate and record the density, ρ_o, of the object and record this in Table 2.
15. Record the accepted value of density (consult your textbook for this value), ρ_{acc}, and record this in Table 2.
16. Calculate the percent error between measured density, ρ_o, and the accepted value, ρ_{acc}, and record this in Table 2.

B. *Density of a Liquid*

17. Use the same object from Part A and, since this was performed in water, the force in air, F_a, in Table 1 and the buoyant force, F_b, in Table 2 will be needed. Transfer this information to Table 3 noting that $F_{bw} = F_b$ from Table 2.
18. Replace the overflow can with the unknown liquid in a beaker. Repeat the procedure in step 7 and record the force, F_{uf}.
19. Use Equations 9 and 10 to calculate the density, ρ_f, and specific gravity, *S.G.*, of the unknown liquid and record these values in Table 3.
20. Use the hydrometer to measure the specific gravity of the unknown liquid and record this as $S.G._h$.
21. Using $S.G._h$ compute the density and record it as ρ_h.
22. Find the percent difference between ρ_f and ρ_h.

C. *Density of a Floating Object*

23. The procedure for this is similar to Part A, but we won't do overflow. Instead use a regularly shaped floating object (rectangular solid, cylinder, etc.). Using your knowledge of the volume of solids, measure and calculate the volume and record this in Table 4. Below Table 4 describe how you calculate this volume and record data taken to perform this calculation.
24. Place the spring scale on the ringstand. Alternatively, if using a lab balance, mount the lab balance on a ringstand about two feet high.
25. Weigh the sinker, m_s, and the floating object, m_o. From Equation 14 we observe that it is not necessary to know the mass of the sinker, but just-in-case and for good measure, we'll record it anyway. Record m_o, calculate the object's weight, F_o, and record it in Table 4.
26. From m_o and V_o calculate the density. Record it in the column ρ_{od} (density object direct measurement).
27. Using thread tie the sinker below the floating object and tie a loop above the floating object.
28. Place a beaker about half full of water on a lab jack under the sinker/object combination.
29. Also for good measure record the density of the fluid, ρ_f. Since you're using water, $\rho_f = 1.00$ gm/cm^3.
30. Record the weight (force in air), F_a, of the sinker/object combination.
31. Hang the sinker/object combination by the short thread from the hook beneath the balance pan (or spring scale), then raise the lab jack until the sinker ONLY is completely submerged in the water. Record this force, F_s.
32. Now raise the lab jack until both the sinker and object are completely submerged, BUT THE SINKER SHOULD NOT BE RESTING ON THE BOTTOM. If necessary, add more water. Record this force, F_u.
33. Use Equation 14 to calculate the density, ρ_o, of the object and record this.
34. Clean up and put away equipment.

Post-Lab Questions – Answer on a separate page and attach

1. In Table 2 you found the buoyant force using two methods. Do your results confirm or contradict Archimedes' Principle? Explain thoroughly.
2. Also in Table 2 you compared your measured density with the accepted density? Do you believe the Archimede's Principle is a good method or poor method to measure density? Explain thoroughly.

3. Due to the string used, will the measured force be greater than or less than the true force? Also due to the string, will the measured volume be greater than or less than the true volume? Combining these two influences will the measured density be greater than or less than the true density or about the same? Explain thoroughly.
4. As a hypothetical example, if you submerge an object with the same density as the fluid, will the object sink, rise, or remain unchanged? Explain.
5. How would you cause an un-tethered, at rest, un-powered submarine to rise? To sink? Are you changing the density, volume, and/or mass of the submarine? How is this different from a hot air balloon? Explain.
6. What is the net force on a floating ice cube? A floating block of wood? A steel bolt floating in mercury? Explain.
7. Which is heavier, 1 cm^3 of ice or 1 cm^3 of water? Explain.
8. If you have a beaker of water and a block of wood balanced on a mass balance and you put the wood in the water, will the mass balance change? Why or why not? Wouldn't the buoyant force on the wood cause the mass balance to show less mass when the wood is in water? Similarly, if pigeons in a truck were flying, would it weigh less? Explain.
9. A lift is rated to lift 10,000 N (about a ton). If it were lifting an aluminum object ($\rho = 2.7$ gm/cm^3) out of a lake, what is the maximum mass of aluminum it can lift? What is the weight of this maximum mass?
10. Explain how a hydrometer works. Consider one with a cross-sectional area of 1 cm^3 and mass of 100 gm. What length is underwater? If it measures alcohol with $\rho = 0.8$ gm/cm^3 how much is below the surface? If it measures antifreeze with $\rho = 1.1$ gm/cm^3 how much is below the surface? Assume it's sufficiently long that it won't sink.

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Table 1 – Raw Data for Density of Sinking Object and Direct Confirmation of Archimedes' Principle

Object Material	m_o ( )	F_a ( )	m_b ( )	F_u ( )	m_{comb} ( )	m_w ( )

Table 2 – Results for Density of Sinking Object and Direct Confirmation of Archimedes' Principle. Note, accepted values of density is tabulated in Appendix A.

Object Material	W_w ( )	F_b ( )	$F\ Err\%$ ( )		ρ_o ( )	ρ_{acc} ( )	$\rho\ Err\%$ ( )

Table 3 – Density of an Unknown Liquid

F_a ( )	F_{bw} ( )	F_{uf} ( )	ρ_f ( )	$S.G.$ ( )	$S.G._h$ ( )	ρ_h ( )	$\rho\ \Delta\%$ ( )

Table 4 – Density of a Floating Object

$m_s =$ ( ), $F_a =$ ( ), $\rho_f =$ ( )

Object Material	V_o ( )	m_o ( )	ρ_{od} ( )	F_o ( )	F_s ( )	F_u ( )	ρ_o ( )	$\rho_o\ Err\ \%$ ( )

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University Physics Workbook Part I
Physical Pendulum (Oscillator)

Purpose

To determine the relationship between the period of a square physical pendulum and the length of its sides and compare to the theory of this type of pendulum.

Theory

A physical pendulum is defined as a rigid body mounted so that it can swing in a vertical plane about an axis passing perpendicularly through it. It differs from a simple pendulum in that it cannot be approximated by a point mass.

The theory of harmonic oscillation predicts that when there is a restoring "force-like" quantity causes an "acceleration-like" quantity proportional to and in the opposite direction as a "displacement-like" quantity then the system will oscillate. Furthermore, the angular frequency of oscillation is equal to the square root of the constant of proportionality. For a spring the "force-like" quantity IS the force and the "displacement-like" quantity IS the displacement. However, even for a simple pendulum, the "force-like" quantity is torque and the "displacement-like" quantity the angle. In electricity the "force-like" quantity is voltage, the "acceleration-like" quantity is change of current divided by change of time, and the "displacement-like" quantity is charge.

Using your knowledge of rotational mechanics from lecture include a discussion of the theory of the physical pendulum and predict the period versus side length in your lab report. For reference, the formula for the moment of inertia around the center of a rectangular plate of dimensions x and y is:

$$I = \frac{M}{12}(x^2 + y^2)$$

When applying this formula do not forget the parallel-axis theorem.

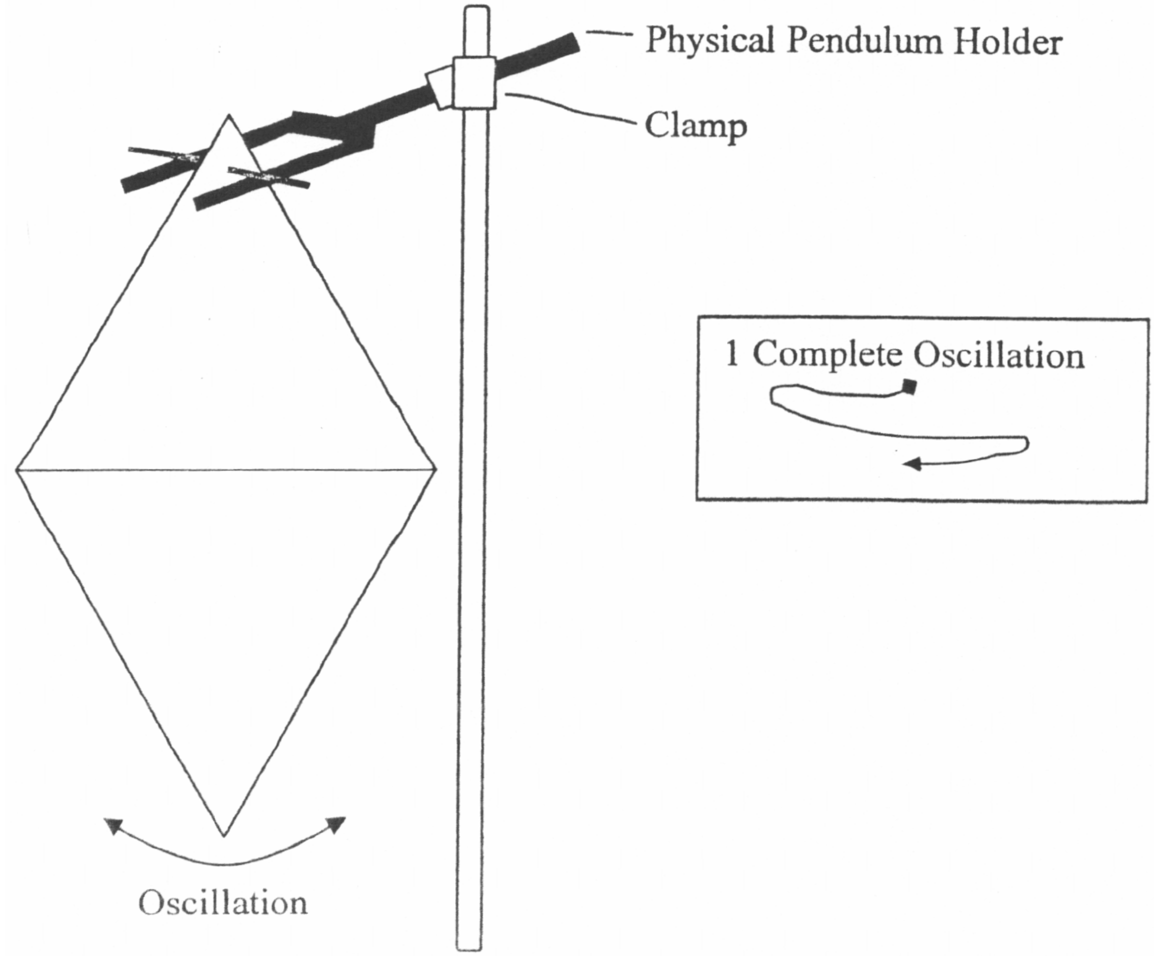
Procedure

You will be provided with several square sheets of metal which will have pins through one corner so that the sheets can oscillate about the pin as shown.

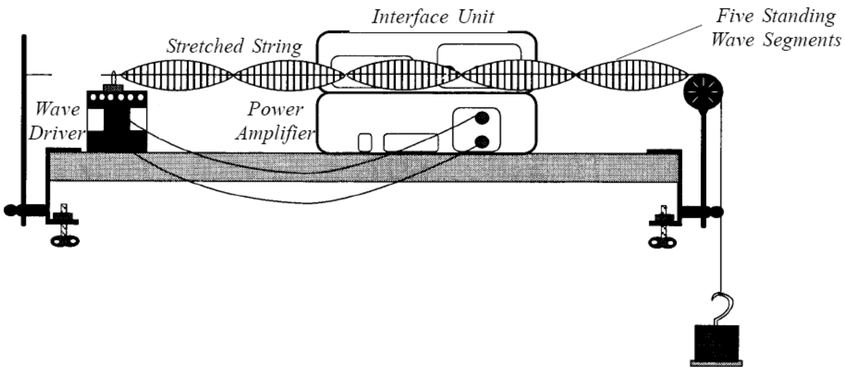
The period of a pendulum is defined as the time it takes the pendulum to make one complete oscillation. The accuracy of measurement of the period can be increased by finding the average time per oscillation. While keeping the angle of oscillation small (less than *15°)*, determine the period of each pendulum after allowing it to complete 20 oscillations.

Graphically determine what relationship exists between the period of the pendulum and the length of its side (Period vs. Length). Plot by hand using a ruler – DO NOT USE A COMPUTER! To confirm the relationship between period and length you may need to make a second graph and/or transform the data.

Diagram Physical Pendulum Holder



University Physics Workbook Part I
String Harmonics (Waves)



Purpose

In this experiment, you will explore the relationship between string length, wavelength, frequency, linear density, and string tension in a standing wave, thus gaining an empirical understanding of the normal modes of vibration in a stretched string.

Equipment

- Frequency generator
- Meter stick
- Pulley
- Banana plug patch cords (multiple)
- Mechanical wave driver
- Clamps (C clamps, table clamps)
- Amplifier (may not be necessary)
- String
- Mass set with hanger (up to 2 kg total)
- Support rod

Introduction

A wave moving within any material is evidence that energy is being transported as the result of a disturbance. There are two distinct categories of waves: mechanical and electromagnetic. Mechanical waves require some kind of material to travel in, but electromagnetic waves, including light, do not.

The speed of both categories of waves depends on two properties of the material they are moving through. For mechanical waves they are an inertial property and an elastic property. For electromagnetic waves they are the permittivity and permeability of the material. For a mechanical wave in a stretched string, the inertial property is its linear density (its mass per unit length), and the elastic property is the tension force in it.

A wave will propagate along the string if you disturb its equilibrium state at any position. When the wave reaches either end, it will reflect and propagate back toward the disturbance.

If you make the disturbance repetitive by using, say, an electric vibrator at one end, the waves propagating away from the vibrator interfere with those that are reflected back from the other end. If the length of the string is an integral multiple of the half-wavelength of the interfering waves, the interference pattern (blur, wave envelope) will be stationary in the string. Such a stationary wave pattern is called a *standing wave*. Using a high speed camera we show actual motion vs. what the eye sees on this YouTube video: http://www.youtube.com/user/SizemoresScience#p/a/u/0/VmGWvuqFqxQ.

In this experiment, you will create standing waves in a stretched string and then measure their wavelength. You will explore the relationship between string length, wavelength, frequency, linear density, and string tension in a standing wave, thus gaining an empirical understanding of the normal modes of vibration in a stretched string.

You will compare your measurements of standing waves to the theory that relates these properties. When you are finished, you will be able to
Explain how standing waves are created.
Identify the nodes and antinodes and the number of segments in a standing wave.
Discuss the factors that determine the natural frequencies of a vibrating string.

Theory

The properties that characterize a wave are its wavelength λ, its frequency of oscillation f (measured in hertz, or 1/s = s^{-1}), and its speed v. These properties are related by the equation:

$\lambda f = v$ ***Equation 1***

Mechanical waves propagate through a medium in either a longitudinal or a transverse mode. In a longitudinal wave, each particle in the medium oscillates in the same direction as the wave propagation. Sound waves in any material travel in this manner. In transverse waves, each particle oscillates perpendicular to the direction of wave propagation. The waves in a stretched string vibrate in a traverse mode.

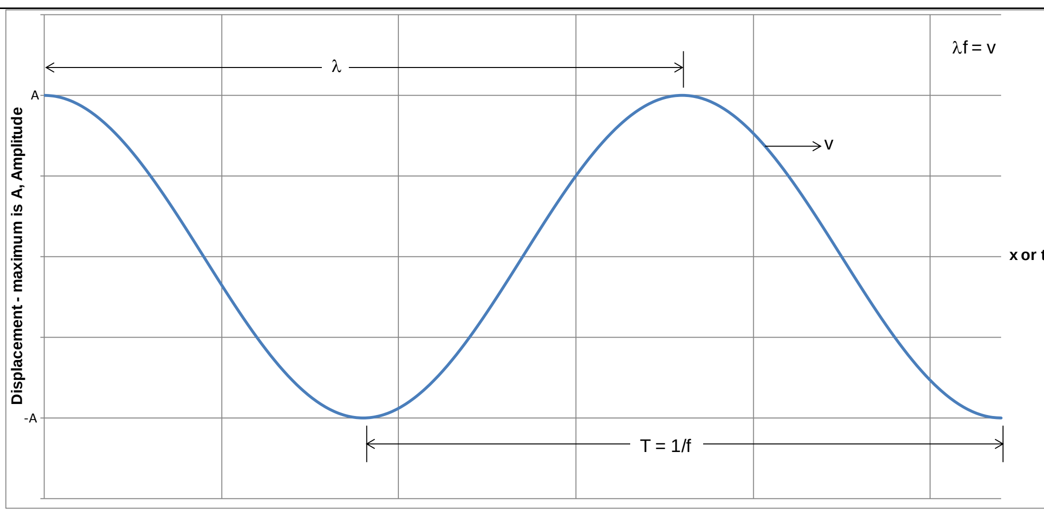


Figure 1 – Plot of wave height (displacement) vs. distance, x, or time, t. Interpreting this as a plot vs. x, this would be a snapshot of the wave at an instant of time. Interpreting this as a plot vs. t, then one would stay at a particular location and record the wave height as it progresses over time.

As each particle oscillates, its maximum displacement up and down is called the wave's *amplitude*, designated as +*A* or –*A*. Figure 1 is a plot of displacement vs. either position or time. The energy being carried by the wave is related to its amplitude. The period of oscillation is inversely related to the frequency:

$T = 1/f$ ***Equation 2***

Let's think about why Equation 1 and 2 are true. A period is the time required for a wave to go from its peak, down to the valley, and back up to the peak again when the observer is standing in one position. To understand this, go to the phet simulation (http://phet.colorado.edu/sims/wave-on-a-string/wave-on-a-string_en.html). Select "no end", "oscillate", zero damping, low frequency, and low tension. Position the computer mouse pointer where one of the green dots is at a maximized and observe the time required for the green dot to go down and back to your mouse pointer. This time is the period, *T*.

Frequency is related to the number of cycles, *n*, (up, down, and back up) observed in a given time, *t*, and is defined as:

$f = n/t$ ***Equatioon***

The faster the wave goes up and down the higher the frequency. The time required for one cycle has the special name of period – the same period as above. When we observe only a single cycle, frequency equals one cycle per the period, or:

$f = 1/T$ ***Equation 4***

It is easy to observe that rearrangement of Equation 4 leads to Equations 2.

The wavelength is the distance a wave travels in one period and has a special Greek letter for it, λ. λ is Greek for l and stands for length as in wavelength. Look at the phet simulation again. You will observe (if it is slowed down sufficiently for your eye to see) that the distance a wave travels in one period is the distance from one peak to the next peak. Wave speed is, therefore:

$v = \lambda/T$ ***Equation 5***

Substituting Equation 2 for *T*, one obtains Equation 1 presented previously.

Two waves meeting each other will interfere. The combined wave they produce is a simple superposition of the two waves. If two waves moving in opposite directions have the same amplitude and frequency, their interference produces a standing wave as shown in Fig. 2. The positions of minimum displacement (destructive interference) are called *nodes*, and the positions of maximum displacement (constructive interference) are called *antinodes*. The length of one segment of the standing wave is equal to one-half its wavelength.

When a string is vibrated at one end, waves traveling from the vibrator interfere with waves reflected from the opposite fixed end. This interference produces a standing wave in the string at specific frequencies that depend on the string's density, tension, and length. If the string is vibrated at multiples of this frequency, standing waves with multiple segments will appear. The lowest frequency (far left in Figure 3) is known as the *first harmonic* and higher frequencies are higher (second, third, etc.) *harmonics* (see Figure 3).

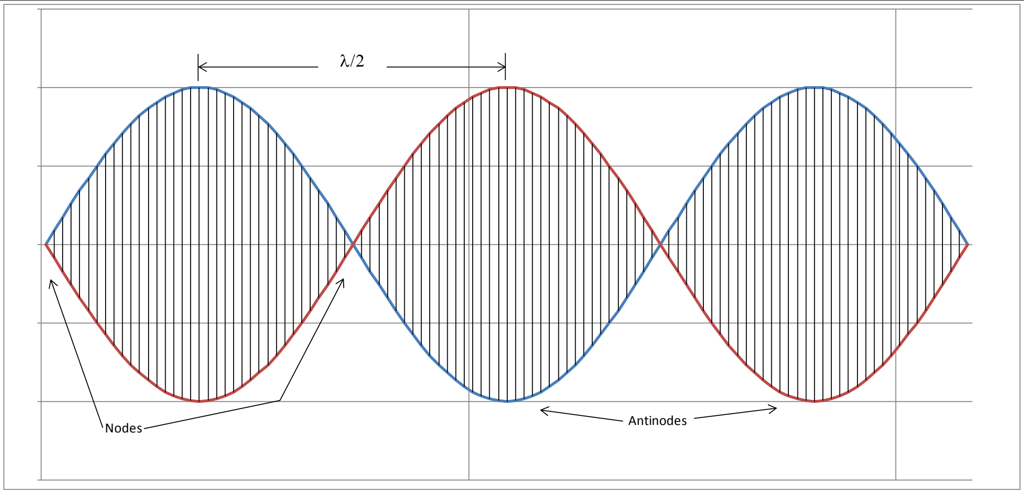


Figure 2 – Standing wave on a string at frequencies higher than the eye can see. The oscillating wave looks like a blur (the vertical lines). At any given point in time the wave can be at the bottom red line, or top blue line, or somewhere in between. See the High Speed Video of a standing wave to observe blurring using ordinary vision, but slow motion from a high speed camera.

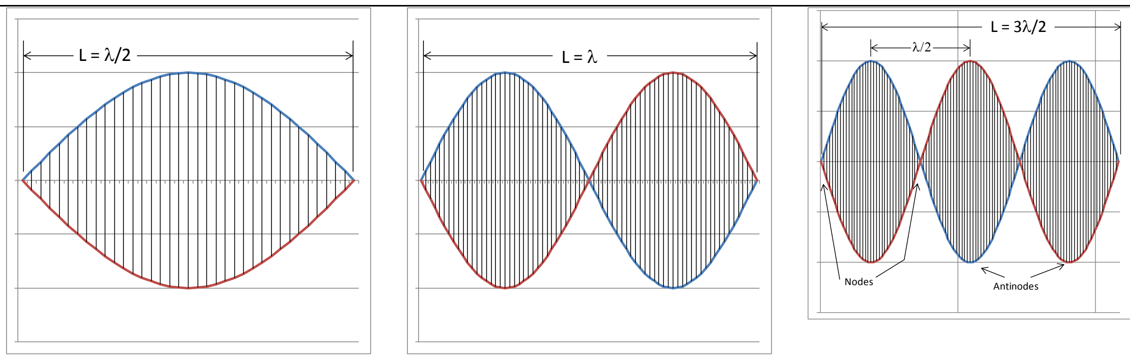


Figure 3 – Standing waves of three different wavelengths on the same string at resonance showing the relationship between resonant frequencies and the strength length.

Note that each segment is equal to one-half of a wavelength. Our vision interprets the rapidly moving string as a blur, but if you video this in slow motion one can observe the entire string oscillating and, furthermore, clearly observe that a segment is one-half wavelength. Thus, for a given harmonic, the wave-length becomes:

$$\lambda = \frac{2L}{n}$$ ***Equation 6***

where L is the string length and n is the number of segments. You can therefore express the velocity of a wave in a stretched string as:

$$v = \frac{2Lf}{n}$$ ***Equation 7***

You can also find the velocity of a wave in a stretched string from the relationship:

$$v = \sqrt{\frac{F_T}{\mu}}$$ ***Equation 8***

where the tension force, F_T, is the force applied to stretch the string and the linear mass density, μ, is an inertial property equal to mass per unit length. You can find the value of μ by weighing a known length of string.

$$\mu = \frac{\text{mass}}{\text{length}} \qquad \textbf{\textit{Equation 9}}$$

Procedure

You will apply tension to a length of string by hanging mass from it over a pulley, as shown in Figure 4. You will then create waves in the string with a computer-driven vibrator and adjust the frequency (Part A) and the tension (in Part B) to create standing waves having from 1 to 7 segments.

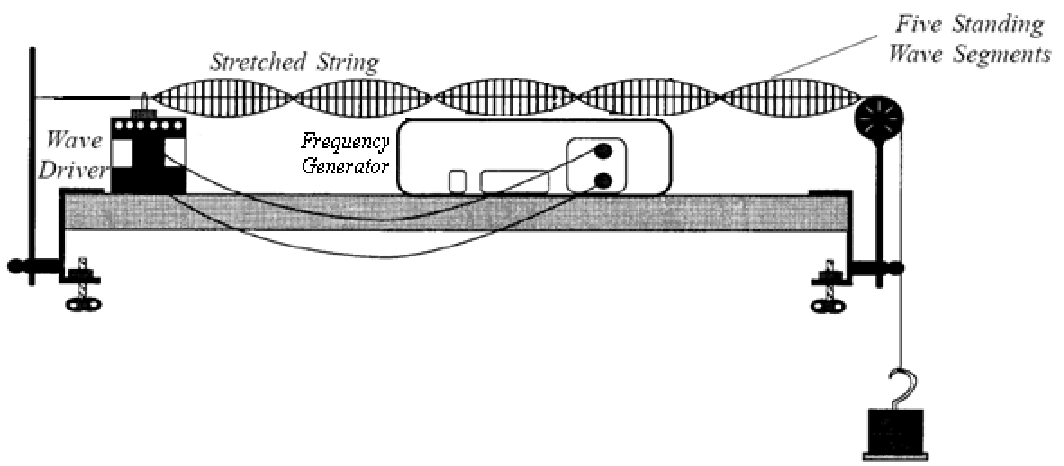


Figure 4 – Schematic of Experimental Apparatus

1) Cut a piece of string about 5 m long. Stretch it out on the table (doubling it if necessary) and measure its length l_s. Wad the string up and carefully measure its mass m_s. Record both values in Data Table 1.
2) Calculate the string's linear density μ and record its value in Table 1.
3) Cut about a 2-m piece of your string and tie a loop in each end. Slip one loop over a vertical support rod that is clamped to the table. Pass the string over a pulley that is clamped to the end of the lab table about 1.5 m away and hook a mass hanger in the other loop.
4) Place the wave driver under the string near the vertical support rod. Slide the loop down the support rod until the string rests in the slot on the top of the wave driver. Use banana-plug patch cords to connect the wave driver to the output of the power amplifier. With the power switched off, plug the Power Amplifier into Analog Channel A. Note that in some experimental arrangements the Power Amplifier is not necessary.
5) Place 1 kg mass on the hanger, however more or less may be required depending on the experimental arrangement.
6) Measure the length of string L between the vibrator and the top of the pulley. Record this length in Table 1.
7) Turn everything on and set amplitude to 1 V (or as near as possible). Vary the frequency until the string vibrates (resonates) at its fundamental frequency (one segment). Adjust the frequency only to obtain the greatest amplitude possible.

8) Vary frequency to get higher and higher harmonics (2, 3, etc. segments) and record resonance frequencies up to 7 segments in Table 1.
9) Add approximately 0.5 kg to the mass hanger and repeat Steps 7 and 8.
10) Add (approximately) an additional 0.5 kg to the mass hanger and repeat Steps 7 and 8.
11) You should have three sets of data – one for each mass.
12) From the frequencies, calculate the periods, T.
13) For each of the three different masses, plot wavelength vs. period for the different harmonics. Find the best fit line and make sure it intercepts zero if it is supposed to. Be sure to write the equation the graph is plotting on the graph and identify what the slope represents. Attach the graphs.
14) Tabulate the slopes, wave speed, of each line from the graphs in Table 2.
15) Compare the measured wave speeds to theoretical wave speed and calculate and enter in Table 2.

Post-Lab Questions – Answer on a separate page and attach

1) In the graphs of wavelength vs. period, what should the slope represent?
2) Should the plot of wavelength vs. period intercept zero? Explain.
3) For each of the three different cases, what were the fundamental frequencies and wavelengths?
4) How many natural frequencies (normal modes) does a simple pendulum have? A mass on a spring? Explain.
5) A violin's E string vibrates at 660 Hz, the distance between clamps is 330 mm, and $\mu = 4 \times 10^{-4}\ \frac{\mathrm{kg}}{\mathrm{m}}$. What is the tension at resonance?
6) What length would the string in Question 5 need to be to resonate at 880 Hz (A about concert A)? How is that length achieved during play by a violinist?
7) What μ would the string in Question 5 require to resonate at concert A (440 Hz)? Is this a thicker or thinner string than the string of Question 5?
8) An tube open at both ends behaves similarly to a standing wave on a string. For the lowest resonance frequency, half a wavelength is the length of the tube, however the open ends are antinodes instead of nodes (like a string). A closed end makes a node. So a tube closed at one end consists of a node and antinode. Express, as a function of the tube's length L, the sequence of resonance wavelengths of a closed tube.
9) A closed tube is 30 cm long and the speed of sound is 343 m/sec. What are the first three harmonics of that tube?

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Lab 13 Report: Names ____________________________________

Table 1 – Data Table Recording Mass, Wavelength, Mass Density, Etc.

String Mass, m_s = ________ (   ), String Length, l_s = ________ (   )

Mass Density, μ = ________ (   ), Distance between string clamps, L = ________ (   )

Mass on Hanger	Tension Force	Theoretical Wave Speed	Harmonic n	Wavelength λ ( )	Frequency f ( )	Period T ( )
M_h = ________ ( )	F_T = ________ ( )	v_t = ________ ( )	1			
			2			
			3			
			4			
			5			
			6			
			7			
M_h = ________ ( )	F_T = ________ ( )	v_t = ________ ( )	1			
			2			
			3			
			4			
			5			
			6			
			7			
M_h = ________ ( )	F_T = ________ ( )	v_t = ________ ( )	1			
			2			
			3			
			4			
			5			
			6			
			7			

Table 2 – Theoretical vs. Measured Wave Speed

Mass ( )	Theoretical Wave Speed, v_t ( )	Measured Wave Speed, v_m ( )	Percent Error

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University Physics Workbook Part I
Specific Heat of Substances

Purpose:
To determine the specific heat of various substances, understand the first law of thermodynamics, energy conversation, and compare this to accepted values.

Factors to be Related

C_s	Specific heat of substance
C_c	Specific heat of calorimeter cup
C_w	Specific heat of water
M_s	Mass of substance
M_c	Mass of calorimeter cup
M_w	Mass of water
T_s	Original temperature of substance
T_w	Original temperature of water
T_f	Final temperature of water and substance

Theory

The specific heat of a substance is the amount of heat necessary to raise the temperature of one gram of the substance one degree Celsius. The heat capacity of a substance is the amount of heat necessary to raise the temperature of a given mass of the substance one degree Celsius. The heat capacity of a substance is M_sC_s. The amount of heat lost by a mass of substance in dropping from one temperature to another is the mass of the substance multiplied by its specific heat and that result multiplied by the difference in temperatures.

To determine the specific heat of a substance, suspend a mass of the substance at a high temperature into a mass of water at a lower temperature. Then determine the final temperature of the water. Applying the law of conservation of energy, the heat lost by the substance is equal to the heat gained by its environment.

The amount of heat lost by the substance is .... $M_sC_s(T_s - T_f)$

The heat gained by the water is .... $M_wC_w(T_f - T_w)$

The amount of heat gained by the calorimeter cup is .... $M_cC_c(T_f - T_w)$

The total measurable heat gained by the environment is..... $M_wC_w(T_f - T_w) + M_cC_c(T_f - T_w)$

From the law of conservation of energy the following equation is found:

$$M_sC_s(T_s - T_f) = M_wC_w(T_f - T_w) + M_cC_c(T_f - T_w)$$

Apparatus

- Calorimeter
- Pot of boiling water
- Substances of unknown specific heat
- Thermometer
- Bunsen burner & stand

Procedure:

1. Start water boiling in pot and bring to vigorous boil. Each lab group may have its own pot and burner OR your lab instructor may set up only one boiling pot for all groups.
2. Find the mass of the substance to be tested (M_s).
3. Suspend substance in boiling water for *5* minutes. Measure the temperature, T_s.
4. Find the mass of the calorimeter cup (M_c).
5. Fill the calorimeter cup half full with water and find the mass again ($M_c + M_w$). Mix cold and warm water to make it a few degrees lower than room temperature.
6. Determine the temperature of the water in the calorimeter (T_w).
7. Quickly remove substance from boiling water and put it into the calorimeter.
8. Find highest temperature to which calorimeter water rises (T_f).
9. Repeat procedure with two or more other substances.

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Team Assessment Form

Date ______ Course ____________ Your Name ____________________________________

Your name is required for credit for one lab – you will be credited with a 100% grade for participation in this survey. Please be as accurate as possible.

Please rate your <u>teammates</u> on the following:

Teammate	___________	___________	___________	___________	___________
Contributions to Team Interaction (choose one for each teammate):					
Interacts poorly, inconsistently, or not at all	☐	☐	☐	☐	☐
Interacts with team members to accomplish the objective	☐	☐	☐	☐	☐
Provides leadership in helping the team accomplish the objective efficiently	☐	☐	☐	☐	☐
Individual Contributions (choose one box for each teammate):					
Is unprepared	☐	☐	☐	☐	☐
Is adequately prepared	☐	☐	☐	☐	☐
Prepared more than was assigned	☐	☐	☐	☐	☐
Contribution to Team Purpose/Goal (choose one box for each teammate):					
Did not contribute, or poor contribution, to lab group	☐	☐	☐	☐	☐
Adequate contribution to lab group	☐	☐	☐	☐	☐
Superior contribution to lab group	☐	☐	☐	☐	☐

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Appendix A: Table of Accepted Densities

Complete Table of Densities (at http://www.periodictable.com/Properties/A/Density.html)

Solids	$\rho \left(\frac{gm}{cm^3}\right)$	S.G. (Specific Gravity) – no units
Gold (Au)	19.3	19.3
Lead (Pb)	11.3	11.3
Silver (Ag)	10.5	10.5
Copper (Cu)	8.9	8.9
Steel (Fe)	7.8	7.8
Tin (Sn)	7.29	7.29
Zinc (Zn)	7.14	7.14
Aluminum (Al)	2.7	2.7
Brass	8.4 – 8.7	8.4 – 8.7
Balsa Wood	0.3	0.3
Oak	0.8	0.8
Earth Average	5.52	5.52
Crown Glass (ordinary, no-lead glass)	2.4 – 2.8	2.4 – 2.8

Liquids & Gases	$\rho \left(\frac{gm}{cm^3}\right)$	S.G. (Specific Gravity) – no units
Mercury (Hg)	13.6	13.6
Water	1.0	1.0
Oil	0.9	0.9
Alcohol	0.8	0.8
Antifreeze	1.125 (32°F) 1.098 (77°F)	1.125 (32°F) 1.098 (77°F)
Air	$1.29*10^{-3}$	$1.29*10^{-3}$
Hydrogen	$9.0*10^{-5}$	$9.0*10^{-5}$
Oxygen	$1.43*10^{-3}$	$1.43*10^{-3}$

Appendix B: Table of Specific Heats

Specific heats for various substances at 20 °C

Substance	c $\left(\frac{\mathrm{J}}{\mathrm{gm\ °K}}\right)$ or $\left(\frac{\mathrm{kJ}}{\mathrm{kg\ °K}}\right)$	c $\left(\frac{\mathrm{cal}}{\mathrm{gm\ °K}}\right)$, $\left(\frac{\mathrm{kcal}}{\mathrm{kg\ °K}}\right)$, or $\left(\frac{\mathrm{BTU}}{\mathrm{lb\ °F}}\right)$
Aluminum	0.900	0.215
Iron (Steel)	0.45	0.108
Copper	0.386	0.0923
Brass	0.380	0.092
Gold	0.126	0.0301
Lead	0.128	0.0305
Silver	0.233	0.0558
Glass	.84	0.20
Water	4.186	1.00
Ice (-10 C)	2.05	0.49
Steam	2.08	0.50
Mercury	0.140	0.033
Alcohol(ethyl)	2.4	0.58

Appendix C: Periodic Table of the Elements

LIGHT METALS

In the periodic table the elements are arranged in order of increasing atomic number. Vertical columns headed by Roman. Numerals are called *Groups*. A horizontal sequence is called a *Period*. The most active elements are at the top right and bottom left of the table. The staggered line (Groups IIIA and VIIA) roughly separates metallic from non-metallic elements.

Group IA includes hydrogen and the *alkali metals*.

Group VIIA includes the *halogens*.

The elements intervening between groups IIAand IIIA are called *transition elements*.

Short vertical columns without Roman numeral headings are called sub-groups.

Groups - Elements within a group have similar properties and contain the same number of electrons in their outside energy shell.

Periods - In a given period the properties of the elements gradually pass from a strong metallic to a strong non-metallic nature, with the last number of a period being an inert gas.

NON METALS

IA	IIA											IIIA	IVA	VA	VIA	VIIA	VIIIA
Hydrogen 1 H 1																	Helium 4.003 He 2
Lithium 6.939 Li 3	Beryllium 9.012 Be 4											Boron 10.811 B 5	Carbon 12.01115 C 6	Nitrogen 14.007 N 7	Oxygen 15.999 O 8	Fluorine 18.998 F 9	Neon 20.183 Ne 10
Sodium 22.99 Na 11	Magnesium 24.312 Mg 12											Aluminum 26.981 Al 13	Silicon 28.086 Si 14	Phosphorus 30.974 P 15	Sulfur 32.064 S 16	Chlorine 35.453 Cl 17	Argon 39.948 Ar 18
Potassium 39.102 K 19	Calcium 40.08 Ca 20	Scandium 44.956 Sc 21	Titanium 47.9 Ti 22	Vanadium 50.942 V 23	Chromium 51.996 Cr 24	Manganese 54.938 Mn 25	Iron 55.847 Fe 26	Cobalt 58.933 Co 27	Nickel 58.71 Ni 28	Copper 63.54 Cu 29	Zinc 65.37 Zn 30	Gallium 69.72 Ga 31	Germanium 72.59 Ge 32	Arsenic 74.922 As 33	Selenium 78.96 Se 34	Bromine 79.909 Br 35	Krypton 83.8 Kr 36
Rubidium 115.17 Rb 37	Strontium 87.62 Sr 38	Yttrium 88.905 Y 39	Zirconium 91.22 Zr 40	Niobium 92.906 Nb 41	Molybdenum 95.94 Mo 42	Technetium 99 Tc 43	Ruthenium 101.07 Ru 44	Rhodium 102.91 Rh 45	Palladium 106.4 Pd 46	Silver 107.87 Ag 47	Cadmium 112.4 Cd 48	Indium 114.82 In 49	Tin 118.69 Sn 50	Antimony 121.75 Sb 51	Tellurium 127.6 Te 52	Iodine 126.9 I 53	Xenon 131.3 Xe 54
Cesium 132.9 Cs 55	Barium 137.34 Ba 56	57-71	Hafnium 178.49 Hf 72	Tantalum 180.95 Ta 73	Tungsten 183.85 W 74	Rhenium 186.21 Re 75	Osmium 190.2 Os 76	Iridium 192.2 Ir 77	Platinum 195.09 Pt 78	Gold 196.97 Au 79	Mercury 200.59 Hg 80	Thallium 204.37 Tl 81	Lead 207.19 Pb 82	Bismuth 200.98 Bi 83	Polonium (210) Po 84	Astatine (210) At 85	Radon (222) Rn 86
Francium 22.3 Fr 87	Radium (226) Ra 88	89- 103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118

Lanthanum 138.91 La 57	Cerium 140.12 Ce 58	Praseodymium 140.91 Pr 59	Neodymium 144.24 Nd 60	Promethium (147) Pm 61	Samarium 150.35 Sm 62	Europium 151.96 Eu 63	Gadolinium 157.25 Gd 64	Terbium 158.92 Tb 65	Dysprosium 162.5 Dy 66	Holmium 164.93 Ho 67	Erbium 167.26 Er 68	Thulium 168.93 Tm 69	Ytterbium 173.04 Yb 70	Lutetium 174.97 Lu 71
Actinium 227 Ac 89	Thorium 232.04 Th 90	Protactinium (231) Pa 91	Uranium 238.03 U 92	Neptunium (237) Np 93	Plutonium (242) Pu 94	Americium (243) Am 95	Curium (247) Cm 96	Berkelium (249) Bk 97	Californium (251) Cf 98	Einsteinium (254) Es 99	Fermium (253) Fm 100	Mendelevium (256) Md 101	Nobelium (254) No 102	Lawrencium (257) Lr 103

University Physics Workbook Part I
Appendix D: Quick Questions

1) Is it faster for a bicycle rider (or motorcyclist) to stop, turn the bicycle instantly 180°, and accelerate up to speed or to ride in a circle? Explain.
2) If you pull backward on a bicycle pedal, which way will the bicycle move? Explain.
3) Racers race 800 km and the leaders are neck and neck racing at 320 km/hr (kph). One develops car trouble after 75 min. and is in the pit 75 min. How fast does the driver have to travel to catch up to the leader? Explain.
4) An airplane takes off and travels 1 hr at 400 kph in one direction. It changes direction and travels 1.5 hr at 300 kph in a second direction. What is the maximum distance from the starting point? What is the minimum distance?
5) Two balls are released. On drops straight down and the other has an initial horizontal velocity. Which one hit the ground first? Explain.
6) A monkey is hanging from a tree and a boy takes a BB gun and aims it straight at the monkey (if you looked down the barrel the monkey would be at the center). At the instant the boy fires, the monkey releases its hold on the tree. Ignoring air resistance, will the BB hit the monkey? Will it make a difference is the the BB gun is aimed up or down at an angle? Explain.
7) On a cliff over the water you are throwing rocks into the water. You give two rocks the same horizontal velocity. The vertical velocity is the same magnitude, but in one case it's up and in the other case it's down. Will the vertically thrown rock have less velocity, the same velocity, or greater velocity when it hits the water? Explain.
8) An ice cube is floating in water. What is the total force acting on it? If you take this to the Moon ($g_{moon} = g/6$) what will be the new mass? New weight? New total force on the ice cube? Explain.
9) What is the total force acting on an airplane in straight and level flight? Explain.
10) How much work is done when you do isometric exercises? How much work do you do swinging an object in a circle? Explain.
11) If a 150 kg lineman tackles a 70 kg kicker, who exerts the larger force? Explain.
12) Two ice skaters (no friction) initially at rest push away from each other. What is the total momentum? Explain.
13) A mile long stretch of iron railroad track is laid with both ends fixed at 20° C. A heat wave comes along with temperatures of 50° C. Young's Modulus = $200*10^9$ N/m^2 and thermal expansion coefficient = $11*10^{-6}$ /° C.

University Physics Workbook Part I
Appendix E: Lecture Worksheets

The following worksheets have questions that may require extra pages. They are designed so that extra pages may be stapled to the worksheet and handed in to the instructor.

Project

Date: ______________ Class:________________ Section (day/time) ________________

Minimum 2, Maximum 5 people per group – when it gets to 6, split the group in half.

Names (write legibly, include last names) _______________________, _______________________,

_______________________, _______________________, _______________________

In-class groups will participate in a project and present a tri-fold display at the end of the semester. This is very much like a K-12 high school science fair project. Research science fair guidelines, and compare to this, to get ideas, but you will be held accountable to meet the following guidelines.

Due to the short length of Summer semesters, Summer classes are exempt from a project requirement. Homework will assume greater importance.

Tri-fold Required (see photo above) at or prior to end of semester. See below for guidelines. Also refer to the science poster, writing in science, and writing in engineering.

Project ideas: Archimede's screw (water pump), Foucault's pendulum (no model - difficult to build), magnetic levitation, or CHOOSE YOUR OWN.

You may have an idea. If you have an idea, discuss it me, your other professors, and/or contact me. I am looking for a step up from the baking soda and vinegar volcano. But can you design a project to use baking soda and vinegar to weigh air?

Potential opportunity some semester:

- Some semesters we have disabled students who need **notetakers**. Being a notetaker satisfies project requirement, however a tri-fold poster is still required and must have science content.

Project requirements:

1. **Progress Reports:**
   (a) Plan: Due about 1/3 way through semester (see syllabus). Your plan must include a timeline and who is doing what tasks. This spreadsheet is provided as an example: spreadsheet.
   Your plan must also include your hypothesis and experiment design. What are you testing?
   (b) Progress Report: Due about 2/3 way through semester (see syllabus). Be honest – are you on track to finish?
2. **Working model** if project is amenable to a working model. Check with me if in doubt.
3. **Tri-fold Poster Guidelines**: A Tri-fold poster is required at last day of class. The library or writing lab can help AND you get ICHA credit for utilizing the writing center. Organization:
   (a) **Title** (include names of group members)
   (b) **Brief description** (~1 paragraphs)
   (c) **Introduction** - more thorough discussion of background, history, & theory. For trifold make these bullet points.

(d) **Experiment** - Describe equipment and procedure. Vary factors and test the outcomes. View labs as examples. Ask similar questions relating to your project. What did it do? For example, if you build a battery, what was the voltage and how long did it run a light bulb.
(e) **Results** - Take the data, analyze it, and try to draw conclusions. In general we report results and not raw data.
(f) **Discuss Results** - What to the results mean or imply? Discuss cause and effect. Did your project meet expectations or not? What would you do differently in the future?
(g) **Brief, Key Conclusions**
(h) Cite all non-original sources using **MLA or APA style**. Pick one or the other (MLA or APA), but be consistent. The library or writing lab can help. Remember, if you cite the source it's research, if you don't it's plagiarism.
(i) Think for yourself. Gather research sources, however contribute your own thoughts and conclusions. If too much of your tri-fold is citations you won't be accused of plagiarism, but the grade may suffer from lack of originality.
(j) Edit ruthlessly. The emphasis of a tri-fold poster is on visual information & completeness yet very concise
(k) Email me an electronic version of your poster text and photos.
(l) NO FLIPPING PAGES. Everything must be visible from the front including group member names & citations.

Grading: Your grade will depend on how well you comply with these requirements.

Sometimes things don't work out as we plan. Your grade, therefore, will not depend on whether the project works. If it doesn't work I'll be looking for a good explanation of what went wrong; a good attempt; a clear explanation of your project, construction, and procedures; valid testing; results and conclusions; and good tri-fold presentation.

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Moon's Motion

Date: ______________ Class:________________ Section (day/time) ________________

Minimum 2, Maximum 5 people per group – when it gets to 6, split the group in half.

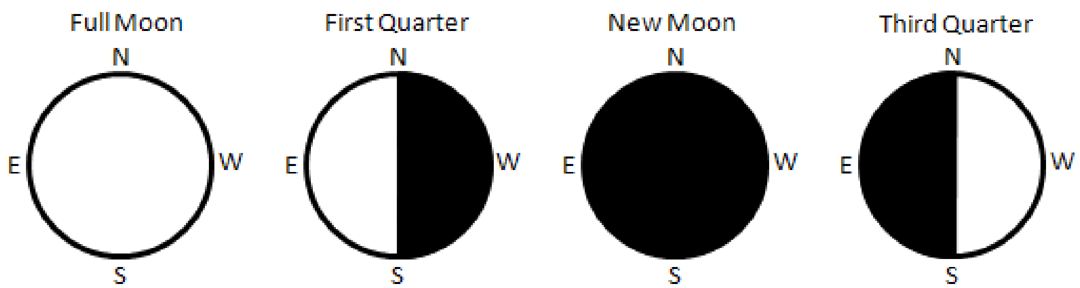
Names (write legibly, include last names) _______________________, _______________________,

_______________________, _______________________, _______________________

The Sun and the Moon are the two astronomical objects that are most evident to us. In this ICA we seek to understand on a basic level how the Moon moves around the Earth and to explain why we reached our conclusion.

Equipment: Your eyes and brains and your teammates

Procedure: Over the period of this class, observe the moon nightly (or daily). Work as a team - if one person forgets to observe, another teammate can pick up the slack. Also for analysis you'll need to discuss this with each other. Sketch the image of the moon similar to below - be sure to label North, South, East, and West. The four most common phases follow:

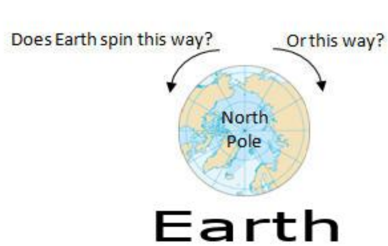


Note that directions are laying on your back with your head to the North looking UP into the sky. In this case East is to the left. If you look DOWN at a map, East is to the right.

Also report Moon rise or Moon set. You may estimate this, for example, if the Moon is halfway up in the sky at sunset, Moonrise was about 6 hr before and Moonset about 6 hr after. Take data for at least one full cycle. Note that about a week before the New Moon the Moon rises just before dawn and is hard to observe at other times. Fill in the best you can.

A calendar is helpful to record your observations – draw the sketch and estimated Moonrise and Moonset in the space for each day of the calendar.

Very Important Analysis: This assignment is incomplete until you can successfully reason, and explain your reasoning *based on your observations*, how and which direction the Moon moves around the Earth. Sketches, like below, and models are very helpful.



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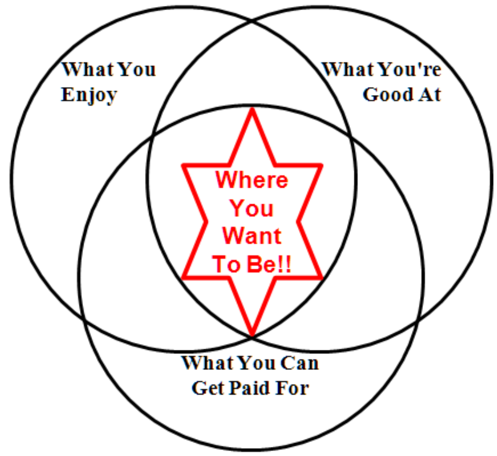
Planning for College

This is an individual take-home assignment. Name: ______________________________

Date: _____________ Class:_______________ Section (day/time) _______________

What are you going to do with the rest of your life and how will college help? Figure out what you enjoy doing, what you're good at, and what you can be paid for. See the figure to the right. The sweet spot, the center, is where you want to go.



Steps To Get There -- GRADUATE!

To graduate, or any other goal, **Plan your work and work your plan!**

Assignment:

Create a *specific* degree plan. For each semester make three columns. The first is the course you will be take, the second column is the number of credits, and the third column is to what degree requirement does the class apply. Does the class apply to your 2 year degree, your 4 year degree, and/or does it apply toward your major?

If you haven't decided on you major, choose the major you are currently leaning towards. If you've partially completed your degree write down what you've done and what you still need to do indicating which is which.

Why make a best guess now if you're undecided. If you plan, now, to travel from Tyler, TX to Los Angeles, CA, and you change your mind deciding to go to San Diego, CA, it's easier to change your destination half-way there than it is to start from Tyler. Just be sure you're not taking the Hasting's Cutoff (the "shortcut" the Donner Party) – try to avoid a drastic change in majors. A few more things:

- If you feel there is something unusual in your plans, write a note about it.
- But be as specific as possible
- You have one week to complete this assignment - it is **due the first meeting of the second week of class.**
- Bug advisors and instructors, search sites (including TJC's, UTT's, or the 4 year college you plan to attend).

Do your homework now - it will save you time, energy, and money!

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Penny Mass (Density)

Date: ______________ Class:________________ Section (day/time) ________________

Minimum 2, Maximum 5 people per group – when it gets to 6, split the group in half.

Names (write legibly, include last names) ________________________, ________________________,

________________________, ________________________, ________________________

Taking accurate measurement is one of the most important aspects of studying physics or, indeed, any other quantitative discipline. You may find these results surprising.

Mass is the amount of matter in an object. Technically, mass is a measure of the inertia of an object and inertia is how much an object resists change in velocity. On the other hand, weight is a special type of force - the force gravity exerts on an object. The mass is the same on the Earth and the Moon, but weight is different because the Moon's gravity attracts less.

Equipment required - pennies, balance, and graph paper

Procedure

1) **Important!** Without anything on the mass balance, adjust it so it reads zero.
2) Carefully find the mass of 10 *different* pennies chosen randomly to the nearest tenth of a gram and record data below
3) Pennies most likely do not have identical masses. To find patterns in data people create a type of graph called a histogram which plots the frequency of each measurement. The vertical (y) axis on a histogram is the frequency and horizontal (x) axis is the mass (increments of a tenth of a gm). Using the graph paper provided (or your own), create a histogram of the mass.
4) Can the pennies be divided into groups based on mass? Explain (attach extra pages if needed).
5) Calculate mean and standard deviation. You are probably familiar with the formula for mean, but standard deviation, a measure of the spread, is probably unfamiliar. The formulas are:

$$\text{Mean} = m = \left(\sum_{i=1}^{N} x_i\right) \Big/ N$$

$$\text{Standard Deviation} = s = \sqrt{\left(\sum_{i=1}^{N} (x_i - m)^2\right) \Big/ (N-1)}$$

Data and Results – create two columns on your paper. Label the first Penny # and the second mass (gm). Record data. On the last row enter your m and s calculations.

Analysis

What do you observe about the mass of the pennies (examine histograms)? Do the pennies look the same? If all the pennies look the same, explain why mass is different? How might you analyze the information differently to provide meaningful information? Perform your revised analysis. Show your results (on a separate sheet of paper if necessary). What do you conclude?

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Roadrunner 

Date: _______________ Class:_________________ Section (day/time) _________________

Minimum 2, Maximum 5 people per group – when it gets to 6, split the group in half.

Names (write legibly, include last names) _________________________, _________________________,

_________________________, _________________________, _________________________

Frequently we will watch a Roadrunner cartoon to find physical law violations in those cartoons. When we do, get with your ICA group, watch the cartoon with the rest of the class, then discuss and list the physical law violations observed. You may use this form and/or a separate piece of paper.

Description of Scene	Name the Law Violated	How was the Law Violated?

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Roadrunner Speed 

Date: _____________ Class:_______________ Section (day/time) _______________

Minimum 2, Maximum 5 people per group – when it gets to 6, split the group in half.

Names (write legibly, include last names) ______________________, ______________________,

______________________, ______________________, ______________________

1) In today's Roadrunner cartoon, in addition to identifying scenes where a physical law is violated (and which law is violated and how it was violated), estimate the Roadrunner's speed. What is your estimate? Show calculations.
2) Based on your estimate, what do you think the problems might be going that fast? Presuming the estimate is less than the speed of light it's possible for an object to go that fast, but other things happen at speeds slower than light speed, but still fast compared to, for example, a jet airplane. What are those "other things" and what might they do to the Roadrunner?
3) Also get with your ICHA group, watch the rest of the cartoon, then discuss and list the physical laws and/or violations. Use a separate piece of paper as needed.

Description of Scene	Name the Law Violated	How was the Law Violated?

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Supergirl's Jump

Date: ______________________

Class:______________________

Section (day/time)____________

Minimum 2, Maximum 5 people per group – when it gets to 6, split the group in half.

Names (write legibly, include last names) ______________________, ______________________,

______________________, ______________________, ______________________

Originally Kryptonites (people from Krypton, that is, Superman, Supergirl, etc.) could only leap - they couldn't fly. Let's figure out how fast Supergirl must take-off to jump to the top of the steeple (21.0 m) or other local towers. The pre-calculus and calculus based classes also have to figure out the acceleration required.

Why did I use the character of Supergirl and not Superman? Women have made great contributions in Science, Technology, Engineering, and Math, but have not always been recognized. Among other things women did the astronomy calculations in nineteenth and early twentieth century, broke the German and Japanese codes in WW II, and discovered splitting of the atom. Also in WW II the armed forces were segregated, but blacks served behind the lines as engineers building road and bridges and other meritorious duties. Science, Technology, Engineering, and Math does not differentiate by race, religion, gender, ethnic origin, or any other differences. EVERYBODY is capable of doing math and science.

1) The first thing to do is figure out how high she must jump. There are several methods to do this, length of shadows, a photo and application of trigonometry, an inclinometer (at exploratorium site - lot's of good stuff there). Since the point of this ICHA is not height measurement your instructor will help with this.
2) After the jump, Supergirl's vertical velocity will decrease at a constant rate (acceleration) given by g (9.8 m/sec^2). First, what is the AVERAGE vertical velocity in terms of time and distance?
3) Second, what is the average vertical velocity in terms of the PEAK velocity (the speed of take-off)
4) Third step: Equate the answers from the first and second question and solve for time in terms of distance and take-off velocity.
5) Fourth step: Using another equation, what is the time in terms of acceleration due to gravity, g, and take-off velocity?
6) Equate the results of Steps three and four and solve for velocity - this should give you velocity in terms of distance and g.
7) Finally, plug in numbers for height and g and solve for the take-off velocity. If you used numbers before this step, you did something wrong.
8) Advanced classes, how would you figure the take-off acceleration? Presume Supergirl can accelerate over a distance of 1 m. Show all your work below (or attach sheets).
9) For calculus based classes, show how to use calculus to figure takeoff velocity and acceleration.

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Treasure Hunt

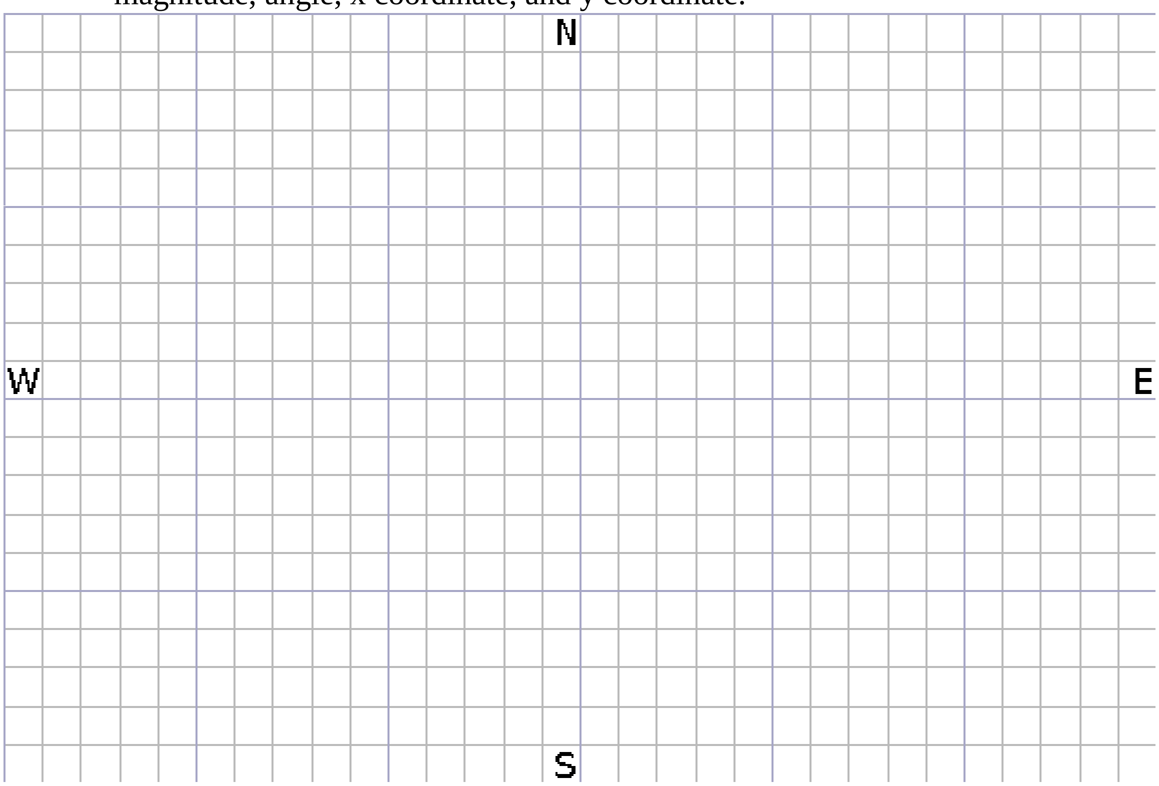
Date: ______________ Class:________________ Section (day/time) ________________

Minimum 2, Maximum 5 people per group – when it gets to 6, split the group in half.

Names (write legibly, include last names) ________________________, ________________________,

________________________, ________________________, ________________________

1) Begin at the bottom (south side) of the sundial. In case of rain your instructor will provide alternative instructions.
2) Take 5 paces (a pace is two steps) south-west.
3) Then take 10 paces south. If you run into flower beds don't tramp through them. Have one group member point south and the other pace the distance, then figure out position at the end.
4) There should be a message at the location. Pick it up and read it.
5) Draw the vectors to scale showing tail-to-tip addition and measure the resulting vector magnitude, angle, x coordinate, and y coordinate.
6) Now run the numbers. Using trigonometry calculate the resulting vector magnitude, angle, x coordinate, and y coordinate.



7) What is the length of the resultant vector? The angle? The x coordinate? The y coordinate? Does this come close to your sketched estimate?
8) Show your calculations. If it matches the paper you picked up, your instructor will give you a "treasure."

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Supergirl's Force

Date: ______________________

Class:______________________

Section (day/time)____________

Minimum 2, Maximum 5 people per group – when it gets to 6, split the group in half.

Names (write legibly, include last names) ______________________, ______________________,

______________________, ______________________, ______________________

1) Originally Kryptonites (people from Krypton, that is, Superman, Supergirl, etc.) could only leap - they couldn't fly. Let's figure out how hard Supergirl must push to jump over TJC chapel steeple (21.0 m high). Last time you calculated how fast she must take-off. What was that value (or recalculate it now)?
2) When she crouches to leap and then pushes, she pushes (uniformly accelerates) over a distance of about 1 m. Derive or find the appropriate equation that applies.
3) What is her acceleration? Show work.
4) What is the equation to find force given mass and acceleration?
5) Supergirl has a mass of 54 kg. How hard does she have to push? That is, what force does she exert to take-off? Show work.
6) Superman has a mass of 101 kg. How hard does he have to push to reach the same speed? Show work.
7) According to DC Comics Supergirl is only 80% as strong as Superman, but let's make our life easier and assume they're equally strong. If they are equally strong, who can leap higher? Why? Explain.
8) Calculus based classes need to go further. Superman's strength is more than 10^{17} N – let's just assume he can exert 10^{17} N force which would make Supergirl's strength 10^{16} N. Compare Superman and Supergirl's final velocity if the push with their maximum force.

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Weighing Air

Date: ____________ Class:______________ Section (day/time) ______________

Minimum 2, Maximum 5 people per group – when it gets to 6, split the group in half.

Names (write legibly, include last names) ____________________, ____________________,

____________________, ____________________, ____________________

Did you know Archimedes invented the raygun? See the picture above - He did! He also figured out how a goldsmith cheated a king. We're going to use the same principle to weigh air.

1) We have evacuated Magdeburg hemisphere's with us today which means it doesn't have any air inside. What's the mass to the nearest tenth gm? Call this quantity m_t (empty - get it) and be sure to list quantities as equations with units, for example, m_t = xxx.x gm (each x represent a digit). m_t is NOT the true mass of the hemispheres. Remember, the mass balance actually measure FORCE exerted on the scale. So what force is exerted in N? Call this f_t.
2) Pressure is force per unit area. One atmosphere pressure is 101,325 N/m^2 at sea level and 15°C. The force holding the hemispheres together is the cross-sectional area times pressure. To get a rough idea how much this is, try pulling the Magdeburg hemisphere's apart. Was it easy or hard? Did it take more than one person?
3) Now that you've pulled it apart (or let the air back in), what is the mass now to the nearest tenth gm? Call this quantity m_a (mass apart). Pick symbols for your quantities that mnemonically remind you of what that quantity represents.
4) What is the weight in N? Call this F_a remembering that weight is just a special type of force. The force of gravity.
5) What is the inside diameter to the nearest mm?
6) Calculate the volume of the hemisphere in m^3. Do you know the formula for the volume of a sphere on the top of your head? This is such a common formula that science students should know it by heart. It is: $V = 4\pi r^3/3$.
7) Archimedes' Principle tells us that the buoyant force equals the weight of the fluid (in this case air) displaced. When the hemispheres are separated there is no buoyant force. Draw force diagrams for our two cases - the hemispheres evacuated and the hemispheres pulled apart. Although Archimedes' deals with fluids, which is in a later chapter, it is introduced when we discuss force because it REALLY helps you understand force balance.
8) From the force balance sketches you just made, derive a formula for the buoyant force in terms of your known quantities. It's a good idea to work things out using scratch paper and putting your final derivation here afterward.
9) Now calculate the buoyant force.
10) Buoyant force is the weight of the air displaced. Calculate the mass of air in kg and density in kg/m^3?
11) The accepted value is 1.28 kg/m^3. Calculate your percent difference. If you're within 30% you're doing well. Why is this measurement so inaccurate?

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Supergirl's Energy

Date: ______________________

Class:______________________

Section (day/time)____________

Minimum 2, Maximum 5 people per group – when it gets to 6, split the group in half.

Names (write legibly, include last names) ______________________, ______________________,

______________________, ______________________, ______________________

1) Originally Kryptonites (people from Krypton, that is, Superman, Supergirl, etc.) could only leap - they couldn't fly. Let's figure out how much energy she needs to jump to the top of the TJC chapel steeple (21.0 m high).
2) If Supergirl is leaping over something else, how tall is it?
3) Last time you calculated how fast she must take-off. What was that value (or recalculate it now on the back of the paper)?
4) What is the equation for kinetic energy?
5) What is her kinetic energy? Her mass is 54 kg. Show work.
6) Energy is not lost or gained. What form of energy does her kinetic energy transform into?
7) What is her gravitational potential energy at the top?
8) From gravitational potential energy, derive a working equation for the height?
9) Calculate the height. Show work.
10) How does this height compare with the height you started with. Is this reasonable? Why? Explain.

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Supergirl's Power

Date: ______________________

Class:______________________

Section (day/time)____________

Minimum 2, Maximum 5 people per group – when it gets to 6, split the group in half.

Names (write legibly, include last names) ______________________, ______________________,

______________________, ______________________, ______________________

1) Last time you calculated the energy it takes Supergirl to jump over the TJC chapel steeple. When she leaps she expends this power is a very short period of time - the time it takes to extend her legs (1 m).
2) If the object Supergirl is leaping over is not 21.0 me (the steeple's height), how tall is it?
3) Last time you calculated how fast she must take-off. What was that value (or recalculate it now on the back of the paper)?
4) Either find or derive the formula for time if you know speed and distance. What is that formula?
5) Now do the calculation to find time. ALWAYS write answer in form of and equation and include units, for example, t = 10.2 sec.
6) Last time you calculated the kinetic energy. What was that value (or recalculate it now on the back of the paper)?
7) From the information in Steps 4 & 5, calculate power.
8) Recalculate power required if she jumped over a 30 m building. 60 m. 90 m, 120 m, etc. Keep going until you see a pattern. Use the SOLVE method one VALID equation per line, keep 3 sig figs & units, express answer as equation with units, & attach extra pages with complete work.

Power at 30 m =

Power at 60 m =

Power at 90 m =

Etc.

Pattern observed?

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Supergirl's Momentum

Date: ______________________

Class:______________________

Section (day/time)____________

Minimum 2, Maximum 5 people per group – when it gets to 6, split the group in half.

Names (write legibly, include last names) ______________________, ______________________,

______________________, ______________________, ______________________

The original Superman couldn't fly – he could only "leap over tall buildings in a single bound." By the time Supergirl came along he and she ould fly. But let's go back to the original, pretend she could only jump, and, this time, figure out how much momentum she needs to jumps to the top of the TJC chapel steeple.

1) If Supergirl is not jumping over the steeple (21.0 m), how high is the object.
2) Last time you calculated how fast she must take-off. What was that value (or recalculate it now on the back of the paper)?
3) What is the equation for momentum?
4) What is her momentum? Her mass is 54 kg. Show work.
5) What is her momentum when she reaches the top?
6) Why did her momentum change? Explain.

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Rocket Report

Date: ______________ Class:________________ Section (day/time) ________________

Minimum 2, Maximum 5 people per group – when it gets to 6, split the group in half.

Names (write legibly, include last names) _______________________, _______________________,

_______________________, _______________________, _______________________

Observe the water rockets, record the information below, and attempt to surmise how to improve the rockets performance.

Rocket	Characteristics	How did it perform? How high did it go?	How might it perform better?

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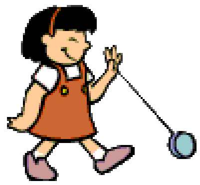
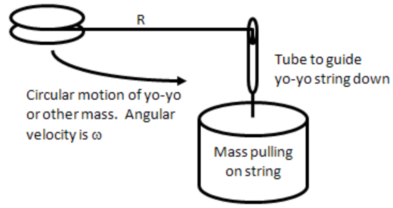
Yo-yo Centripetal Force





Date: _____________ Class:_______________ Section (day/time) _______________

Minimum 2, Maximum 5 people per group – when it gets to 6, split the group in half.

Names (write legibly, include last names) ______________________, ______________________,

______________________, ______________________, ______________________

1) When twirling a yo-yo (or any mass on the end of a string) in a circle, there is a centripetal force - the tension in the string. Our goal it to calculate it and compare it to the weight creating the centripetal force.
2) What is the formula for centripetal force?
3) What is the mass of the yo-yo (Instructor supplied)? We'll call this m_y.
4) How long (R) is the string from the yo-yo (or other mass) to the center of rotation.
5) Use a timer to determine the angular velocity, ω, of the yo-yo. A good way to do this is count 10 revolutions and find the time. Enter that here:
6) What is the time for one revolution, T?
7) Remember, angular velocity is the number of radians per second and there are 2π radians per revolution. So the formula for angular velocity is $2\pi / T$ in units of radians per second. Calculate and enter the angular velocity in radians per second.
8) What is the formula for the speed the yo-yo is moving?
9) Calculate the speed.
10) Using the formula for centripetal force, calculate the centripetal force.
11) The mass pulling the string, m_p, is what value?
12) What weight does mp exert?
13) Compare this weight to the centripetal force. Are they close? Why or why not? Explain.

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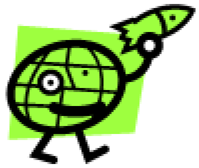
Moon Guns



Date: ______________ Class:________________ Section (day/time) ________________

Minimum 2, Maximum 5 people per group – when it gets to 6, split the group in half.

Names (write legibly, include last names) _______________________, _______________________,

_______________________, _______________________, _______________________

If homo sapiens (us) every get to the point of waging war on the Moon, it would only be necessary to pierce the armor of the habitats of vehicles of the enemy. Then the occupants would die of asphyxiation.

Armor piercing rounds have typical velocities of 1400 m/sec to 1900 m/sec.

1) Why would it be a bad idea to fire some of those rounds (the faster ones) on the Moon? Hint: Calculate the circular orbit velocity and escape velocity from the Moon. $G = 6.67428*10^{-11}$ N m^2/kg^2, $R_{moon} = 1.737*10^6$ m, and $M_{moon} = 7.3477*10^{22}$ kg.
2) What is the maximum velocity round you could safely fire on the Moon? Show all your work.

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Rolling Race

Date: _____________ Class:_______________ Section (day/time) _______________

Minimum 2, Maximum 5 people per group – when it gets to 6, split the group in half.

Names (write legibly, include last names) ______________________, ______________________,

______________________, ______________________, ______________________

How many of participated in a pinewood derby in Girl Scouts, Cub Scouts, Brownies, or some other organization? In this competition you built a small model car and rolled it down a ramp. The fastest car wins. We're going to do something similar. We're going to roll several objects down a ramp and see which one goes fastest. Record the objects being raced and their characteristics. Are they spherical, cylindrical, etc? Solid or Hollow? Etc.

Object 1 & Characteristics	Object 2 & Characteristics	Who won?	Why?

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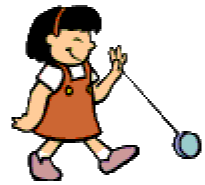
Yo-yo Torqure Force



Date: ______________ Class:________________ Section (day/time) ________________

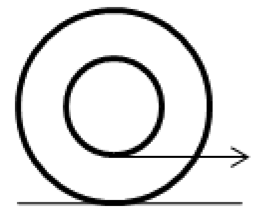
Minimum 2, Maximum 5 people per group – when it gets to 6, split the group in half.

Names (write legibly, include last names) _______________________, _______________________,

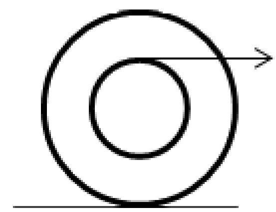
_______________________, _______________________, _______________________

My yo-yo has a spool diameter of _____________ and outer diameter of ______________.

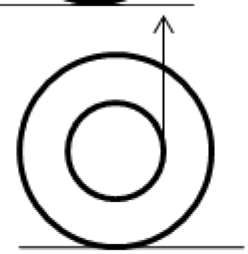
1) What direction will it move when I pull it to the right as shown in the figure at right?



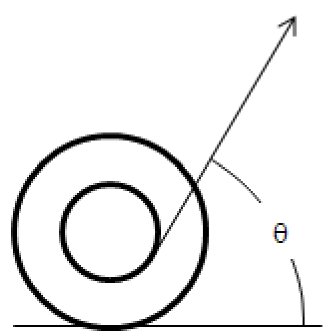
2) What direction will it move when I pull it to the right again, but as shown in the next figure?



3) What direction will it move when I pull it up as shown in the figure below?



4) What the critical angle, θ, where it won't move? What direction will it move if the angle is decreased slightly? What direction will it move if the angle is increased slightly?



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Supergirl's Breaks the Steeple

Date: ______________________

Class:______________________

Section (day/time)____________

Minimum 2, Maximum 5 people per group – when it gets to 6, split the group in half.

Names (write legibly, include last names) ______________________, ______________________,

______________________, ______________________, ______________________

Today we're going to figure out how hard Supergirl must push to break the steeple. You will recall it is 21.0 m high.

1) First, proceed to steeple to measure the area of the legs supporting it.
2) What is the length and width of a column supporting the steeple? Be sure to express values as equation with units with one equation per line. Attach extra pages if needed.
3) Calculate the total area of columns supporting the steeple? Show all work.
4) Bricks and mortar (the clock tower) and concrete have about the same properties - Young's modulus of $20*10^9$ Pa and strength (maximum stress in compression) of $20*10^6$ Pa. You know the area and the maximum stress. Solve for and calculate the force required to break the clock tower. Use the SOLVE method, keep 3 sig figs and units, one equation per line (use extra paper if needed), & write answer as equation.
5) What will be the change in height of the clock tower immediately prior to breaking? Keep using our standard method of problem solving per Item 3.
6) The maximum stress in tension is smaller: $2*10^6$ Pa. Repeat the calculations of force and change in height if Supergirl tries to pull the clock tower apart.
7) How many ordinary, non-Kryptonian, people would it require to perform this feat without pulleys or other machines? A rough estimate of the force a normal person can exert is about 1000 N.
8) Is the answer to Item 6 sensible? Is that about what you expected before doing calculations?

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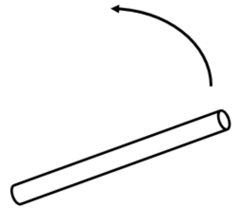
Sound Resonance



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Minimum 2, Maximum 5 people per group – when it gets to 6, split the group in half.

Names (write legibly, include last names) _______________________, _______________________,

_______________________, _______________________, _______________________

Today we're going to swirl a sound resonance tube and calculate the sound resonances.

1) How long is the tube in m?
2) What are the first 4 longest wavelengths that fit in this tube for a resonating sound wave? By this point in the semester I should not have to remind you about the SOLVE method and keeping 3 sig figs and units.
3) The speed of sound is near 340 m/sec. What are the first 4 lowest frequencies?
4) Does this make sense? Below is a list of musical notes and frequencies.
5) ____

Note	f (Hz)	Next octave	Note	f (Hz)	Next octave	Note	f (Hz)
A	110		A	220	Concert	A	440
A sharp/B flat	117		A sharp/B flat	233		A sharp/B flat	466
B	123		B	247		B	494
C	131	Middle	C	262		C	523
C sharp/D flat	139		C sharp/D flat	277		C sharp/D flat	554
D	147		D	294		D	587
D sharp/E flat	156		D sharp/E flat	311		D sharp/E flat	622
E	165		E	330		E	659
F	175		F	349		F	698
F#/G flat	185		F#/G flat	370		F#/G flat	740
G	196		G	392		G	784
G#/A flat	208		G#/A flat	415		G#/A flat	831
A	220	Concert	A	440		A	880

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Fluids

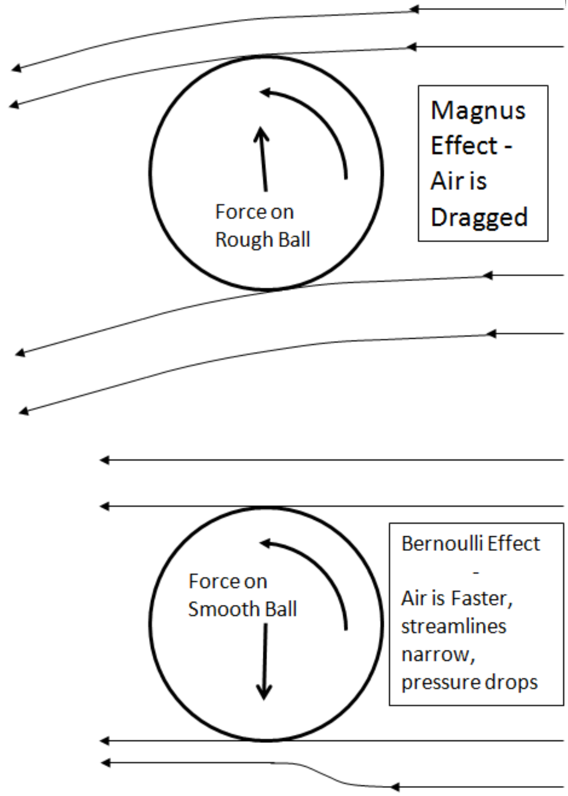
Date: ______________ Class:________________ Section (day/time) ________________

Minimum 2, Maximum 5 people per group – when it gets to 6, split the group in half.

Names (write legibly, include last names) _______________________, _______________________,

_______________________, _______________________, _______________________

1) Take two pieces of ordinary paper and hold them hanging down and loose per the figure at the right. Then blow between them as shown. Do they attract, repel, or do nothing? Explain what is happening. What you are observing is called the Bernoulli effect. Observe a water faucet (or pouring water out of a bottle). First, turn it on slowly so it comes out smooth (this is called laminar flow). What happens to the flow as it speeds up from gravity? Does it narrow or expand? As fluids move faster they become turbulent (turbulent flow). What changes happen as you look down the stream? Describe and sketch this stream of water.The Bernoulli effect ignores drag, however drag is important in many fluids. Due to high drag, baseballs behave just the opposite of what the Bernoulli effect predicts as shown below right. Baseballs curve due to a drag effect - the Magnus effect. Drag can be observed with eggs since raw eggs are a hard shell filled with a viscous fluid while a hard boiled egg is completely solid. The viscous fluid interior should slow the spin of the raw egg. Spin the two eggs, observe, and record your observations below. At the end we will reveal which egg is hard boiled and which is raw. Did you predict this correctly? How did the raw egg behave? The hard boiled egg?



To test this your instructor will choose the hard boiled egg and break it on the instructor's head.

PS - a large number of physics textbooks erroneously state the curve of a baseball is due to the Bernoulli effect.

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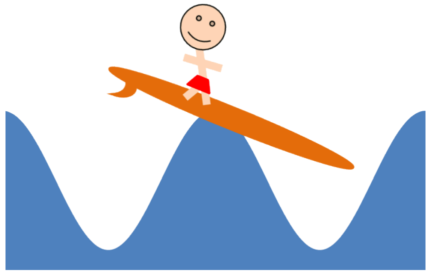
Slinky Wave



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_______________________, _______________________, _______________________

Also refer to slinky waves for conceptual classes.

1) Use a slinky to make a transverse standing wave. Measure the wavelength of the slinky wave. What is it?
2) Use a timer to find the period of a slinky wave. This often works well by timing 10 oscillations and then dividing by 10. What is the period?
3) What formula do you use to find the wave speed?
4) Evaluate the wave speed.
5) Your instructor will provide the total mass of the slinky. What is it?
6) What is the length of the slinky?
7) Combining Parts 5 and 6, what is μ, the mass per unit length?
8) You instructor is going to put a mass on the slinky or use a spring scale to stretch the slinky to find the spring constant. What is the mass or force reading on the spring scale?
9) How much did the slinky elongate?
10) Combining Parts 8 and 9, what is the spring constant k?
11) How are you going to get the tension, F_T, of the slinky? Find F_T.
12) What is the formula for wave speed?
13) Evaluate wave speed using the formula in Question 12.
14) How does the answer to Part 13 compare to the answer to Part 4?

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Doppler

Date: _____________ Class:_______________ Section (day/time) _______________

Minimum 2, Maximum 5 people per group – when it gets to 6, split the group in half.

Names (write legibly,
include last names) ______________________, ______________________,

______________________, ______________________, ______________________

The Doppler effect is so intimately connected to physics that the Society of Physics Students uses it in their logo.

We're all familiar with the fact that when a fire truck or police car goes by it starts with a high pitch sound as it approaches, then it becomes lower pitched after it passes. This is the Doppler Effect.

The Doppler Effect ONLY regards pitch, not loudness. Loudness is mostly determined by its power and how near or how far way a sound source is. We're going to throw Doppler Footballs or some similar device to observe this effect.

1) Now, let's do a problem. Have a friend (pretend) swing sound source that emits a pure 440 Hz A note when at rest. Swing it in a circle. Part of the circle it is coming toward you at 10 m/s, and part of the circle it is moving away at 10 m/s. Draw two sketches – one for the source receding and one for the source approaching.
2) What formulas would you use?
3) What is the wavelength and frequency for the source as it's approaching you? Is the frequency higher or lower?
4) What is the wavelength and frequency for the source as it's receding from you? Is the frequency higher or lower?
5) Now redo this problem if you are moving away from the center of the circle at 2 m/s. Presume the size of the circle is small, in other words, at one point the source is moving toward you at 10 m/s and you're moving away at 2 m/s. At another point the source is moving away from you at 10 m/s and you're also moving away at 2 m/s.
6) What is the wavelength and what frequency *do you hear* for the source as it's approaching you? Is the frequency higher or lower?
7) What is the wavelength and what frequency *do you hear* for the source as it's receding you? Is the frequency higher or lower?
8) Now, think of this problem if the observer is standing still again, but there's a 2 m/s wind from the observer to the sound source.
9) What is the wavelength and what frequency *do you hear* for the source as it's approaching you? Is the frequency higher or lower?
10) What is the wavelength and what frequency *do you hear* for the source as it's receding you? Is the frequency higher or lower?
11) What do you observe about your answers to 7, 8, 10, and 11. Remember all velocities are *relative* to the medium the wave is traveling in. The only exception is electro-magnetic waves and that peculiarity is the basis of Einstein's Theory of Relativity.

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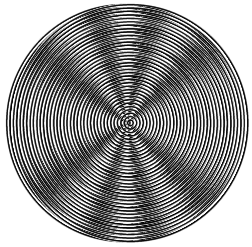
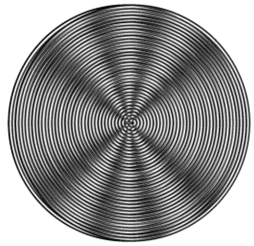
Interference

Date: _____________ Class:_______________ Section (day/time) _______________

Minimum 2, Maximum 5 people per group – when it gets to 6, split the group in half.

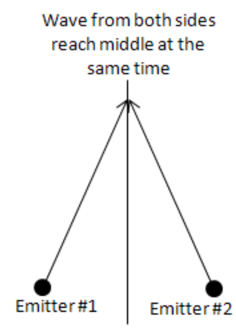
Names (write legibly, include last names) ______________________, ______________________,

______________________, ______________________, ______________________

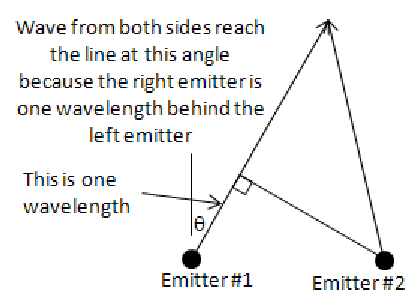
In this activity we will bounce a laser beam (670 nm) at a blank CD and determine the spacing of tracks on a CD. Alternatively use a diffraction grating.

The two images above are like a snapshot of a ripple tank where two wave emitters are two wavelengths apart (the emitters are horizontal). This is like a CD with a spacing of twice the wavelength of the light. Each stripe acts like the wave generator in a ripple tank. Unlike the above image a CD has thousands in a row. But the angle where waves reinforce (constructive interference) are the same. The light areas are where crests and troughs of the two emitters waves go together and dark regions are where the crest of one meets the trough of the other emitter. Let's look at the wave pattern first and try to decide what we expect before we measure the CD.

The first thing to notice is that the right and left pattern above appear identical, but they are not. The waves are 180° out of phase. In the image at left the center is a crest. In the image at right the center is a trough. But reinforcement occurs at the same angles either way.



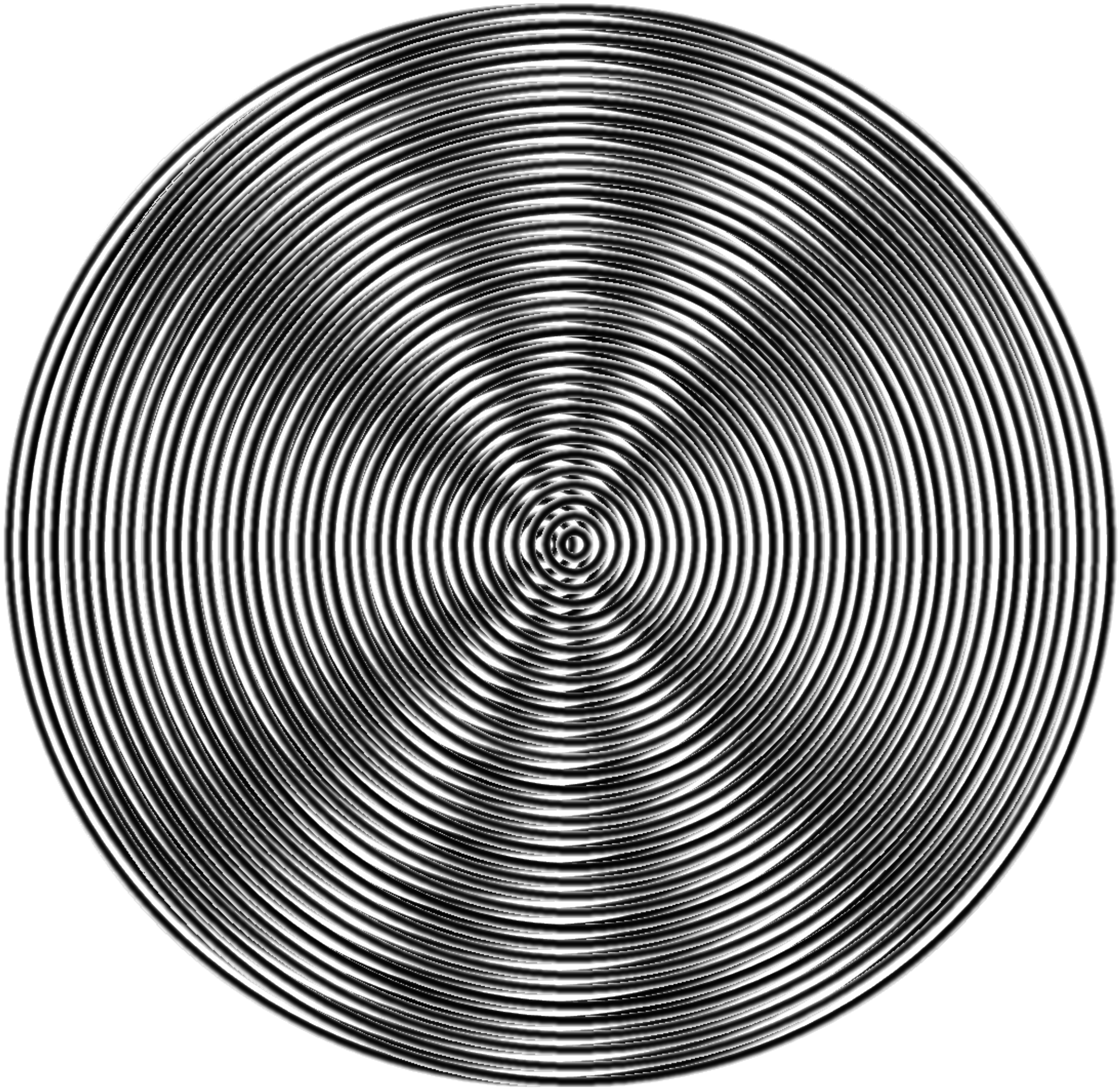
Why reinforcement occurs in the center should be obvious (see figure at right). At any point on a vertical line halfway between the two emitters the wave emanating from each emitter is going to reach that middle line at the same time. So the perpendicular from the line connecting the emitters is going to be where constructive interference occurs.



It's the other angles that are a little bit more difficult to figure out, but not really that hard. The first angle (other than straight ahead) where constructive interference occurs is where there is exactly one wavelength difference between the path lengths as shown in the next figure. The other, larger, angles occur where the path length difference is 2, 3, 4, etc. wavelength difference. The figure at right shows the angle for the first order (1 wavelength difference). The angle, θ, from the center is found from simple trigonometry.

On the next page the left hand top figure is expanded. Using a protractor (or ruler and trigonometry), find the angles of all orders of reinforcement. What are they?

1) Use the symbol d to represent the distance between emitters and use trigonometry to solve for this angles of the orders. What is the formula?
2) If d is two wavelengths, evaluate this formula for all orders. What is θ_1, θ_2, θ_3, etc.?
3) Compare the results of Part 1 and 2. Are they close to each other? Why or why not? Explain.



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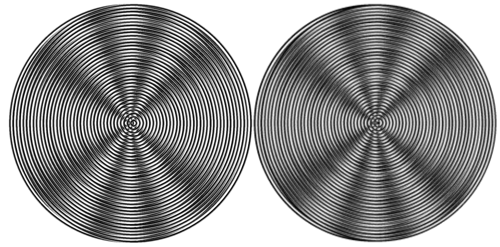
Interference Part II



Date: ______________ Class:________________ Section (day/time) ________________

Minimum 2, Maximum 5 people per group – when it gets to 6, split the group in half.

Names (write legibly, include last names) ________________________, ________________________,

________________________, ________________________, ________________________

Review the previous assignment. You compared the predicted and measured values of the angles where constructive interference occurred. Now consider the CD or diffraction grating. Let's figure out the spacing. The accepted value for a CD is 1.6 μm and the accepted value for a DVD is 0.74 μm. Your instructor will provide the accepted value if you use something else.

1) Use trigonometry to measure the angles where constructive interference occurs as follows: How far from the screen was the CD?
2) How far from the central spot was the first order spot? The second order spot? The third order spot? Etc.
3) Using trigonometry, what is the formula for θ_n given the results of Parts 5 and 6?
4) Evaluate the angles θ_1, θ_2, θ_3, etc. What are they? Write them down – these are your measured values.
5) If you know the angle of a given order, n, how do you find the spacing, d?
6) Calculate and tabulate the spacing for all orders. Is d always the same? Why or why not? Explain.
7) Compare d to the accepted value. How close did you get?

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Heat

Date: ______________ Class:________________ Section (day/time) ________________

Minimum 2, Maximum 5 people per group – when it gets to 6, split the group in half.

Names (write legibly, include last names) ________________________, ________________________,

________________________, ________________________, ________________________

Mechanical energy is easily converted to heat, but the reverse is harder. Today we're going to use thermal probes to understand how mechanical energy is turned into heat.

1. Without touching the probe to anything, what is the temperature in °F? In °C?
2. Hold it in your hand. What is your temperature in °F? In °C?
3. Don't put this under your tongue, however, why do doctors put the thermometer under your tongue (or elsewhere) to measure body temperature?
4. Rub your hands together then hold the thermal probe in your hand. What is the new temperature in °F? In °C?
5. Is your hand hotter now? Why? Or why not? Explain.
6. Measure and record the temperature of several things around the room. The window, the computer, etc. Record your observations. Explain why one thing is warmer than another.
7. What is heat? Why can energy be converted to heat, but not all heat can be converted back to useful energy? Does this mean energy isn't conserved? Why or why not? Explain.

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Entropy

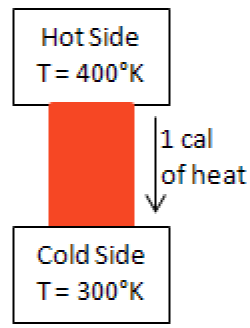
Date: ______________ Class:________________ Section (day/time) ________________

Minimum 2, Maximum 5 people per group – when it gets to 6, split the group in half.

Names (write legibly, include last names) ________________________, ________________________,

________________________, ________________________, ________________________

When something gains heat it becomes more disordered and it's hard to get that order back. The ice cube above, for example, gains heat and becomes a disordered puddle of water. Going back to become an ice cube is more difficult.



Watch this video (it helps to turn the poor soundtrack off): http://www.youtube.com/watch?v=D-y73F0A02Y. Can you tell when the film switches from running forward to running backward? What finally informs you that the film was running backward?Let's solve an entropy problem. If 1 cal of heat enters the hot side of a copper heat conductor at 400°K and leaves cold side at 300°K, calculate the change in entropy. First, how much entropy does the hot side lose?

3) How much entropy does the cold side gain?
4) Add the answers from Parts 2 and 3 - What is the total entropy change?
5) The second law of thermodynamics says entropy increases. Did this happen?

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Appendix F: Study Habits & Work Ethic

Seven Habits of Successful Students

1. **ATTEND ON TIME – ALWAYS!** Don't miss except for illness, hardship, or other emergency. Learn what work you missed and make it up. Arrive on-time, take notes, and make the best use of your time in class.
2. **2-3 hours of study outside class for every contact hour in class** is normal and expected. This is the norm and ill prepared students will need to do more.
3. **Finding and fixing mistakes is more important than never making a mistake.** We're only human – error elimination is impossible. The 3 main ways to find and fix mistakes are step-by-step organized written problem solving (SOLVE), estimation, and dimensional analysis.

- **Use the SOLVE method** (or roughly equivalent method). SOLVE stands for **Sketch** to understand problem, **Organize** (list known and unknown quantities), **List** appropriate equations, **Vary** equations step-by-step in an organized fashion and write down each step on paper line-by-line to symbolically find the unknown quantity**,** **Estimate/Evaluate** – estimate a ballpark for your answer and then plug in numbers at the very end of the process. Compare your estimate to the result to decide if your result is reasonable.

  If the estimate and result are not comparable then there was probably a mistake somewhere. Review your step-by-step written work to attempt to find the mistake.

  There are other methods you may have learned. As long as you are consistent in the use or an organized method, I will award full credit. Note that all these methods employ two find-and-fix techniques to find-and-fix mistakes.
- **ALWAYS use dimensions, dimensional analysis, and convert units using fraction multiplication** (or solve a proportion). NEVER CONVERT UNITS IN YOUR HEAD – this commonly leads people to multiply when they should divide and divide when they should multiply. ALWAYS write your conversion using fraction multiplication or, less efficiently, proportions. REVIEW YOUR WORK if dimensions don't agree – there is probably a mistake somewhere.

4. **Keep 3 sig figs ALWAYS and sometimes more**. Never drop below calculating results to 3 sig figs. Sometimes more are needed for answers accurate to 3 sig figs. In future courses this rule will be refined, but for now you will be graded down if you fail to keep at least 3 sig figs.
5. **Always take lecture notes**. We learn by taking notes even if we throw them away at the end of lecture. But don't toss the lecture notes – do the next step.
6. **Continually summarize material**. There are several summary methods – Cornell, SQ3R, etc. Summarizing commits information to long-term memory. Habitual, e.g. daily, summary reinforces memory each time you do it. Pick a method and use it consistently. In Prof. Sizemore's classes he allows a single summary card on exams.
7. **Continually do and UNDERSTAND homework**. Getting the correct answer at the end is less important than learning and understanding the concepts and the process used to arrive at the correct answer. You may get lucky getting the correct answer, but understanding best prepares you.

For your amusement, here's an article on **How to Succeed at Failing.**

How to Succeed at Failing College

A guide for the perennial (SAT word) slacker

Jonathan Lavigne
Ebbtide Staff Slacker

Here are twelve ways to make sure that your time in Shoreline Community College (SCC) is time well wasted indeed. For those who think that cutting class and never showing up on time are too cliché, follow these simple steps and you will be on your way to earning your degree in the next 10 to 12 years.

12. Do other things full time.

Buy yourself a used car and fix her up. You need gas so get a part-time job flipping burgers during the grave yard shift. With whatever extra money you have left over, take your girlfriend out to extravagant places on week-ends.

11. Skip your classes to finish other assignments.

You start by not doing your assignments in advance because you need that extra "pressure" to produce. So when you are late, you skip math to do that English essay. Then you skip English to do your physics proj-ect, etc. . .

10. Study only when you like the teacher.

Because in life, we can't just do good work for anybody. You need to have a good rapport with the teacher before you can give him what he wants. And when you hate the teacher you only end up giving him second rate material anyway.

9. Skipping tests if you didn't study.

After trying this one a few times, calculate how much you need on your final to get a pass-ing grade. When you figure out you need 130% to pass the class, resign yourself to try agair next quarter. Unless math is the class and you think you still have a shot at it.

8. Don't ask, don't tell.

You don't understand some-thing? Nobody else probably does. I'm sure someone else will ask, before he moves aiong to something else. And wha. if you ask and sound stupid? Better stay quiet.

7. Study only when you are ready

Sometimes you need to be inspired to study, you have to be in the right mood. Hold on till you get there.

6. Beg the teacher for exten-sions

They love that, they really do. The hardest part is coming up with an adequate story that is believable enough not to require coming to class with a cast on your head claiming you have a tumor. Bonus point for trying it in a biology class.

5. Never submit your work until it's perfect.

It might take a week, it might take a month, but it's all or noth-ing.

4. The "Hey, if it worked before..."

You sailed your way right through high school and didn't even crack open that geometry book didn't you? Why change now? It worked for you then why wouldn't it work now?

3. The "Brokenhearted" syndrome.

Like all love stories, math should be involved. For every three months you have been together, it takes one month to get over it. So if you were with him/her since this summer, you have till three days after memo-rial day to get over it. Don't let anything as trivial as homework or assignments get in your way, you have a broken heart to mend.

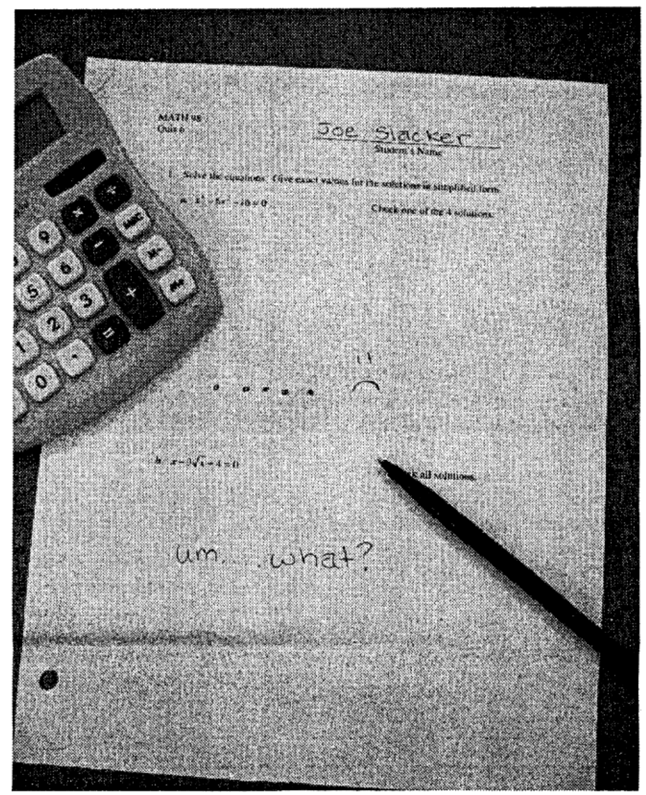
2. The Diet Technique

Study 18 hours a day. Take two 10-minute breaks for food a day. NOTHING MORE! Add that up with a 1000 calorie diet your sure to look your ghostliest by graduation day! Of course your grades might be phenome-nal in October but once the end of the year comes around, you'll see how low they can go.

1. The Brain Wash Method.

Get a mirror. Look at yourself in the mirror and repeat after me: "I can't do this. I'm going to fail. I don't understand. It's too hard for me. Why am I doing this..." Do these steps once a day in a daily affirmations kind of way.

I would hope that you took notes on all that but you proba-bly dozed off before you read the word "Here." So maybe you didn't need my help after all.



University Physics Workbook Part I
Appendix G: Teamwork & Problems

For individual (and study group) quantitative problem solving we strongly, strongly encourage the use of the SOLVE 5-Step Problem Solving method.

1. **S – Sketch** Sketch, draw a picture, understand the problem
2. **O – Organize** Organize, write down known and unknown quantities
3. **L – List** List relevant equations, determine which are applicable
4. **V – Vary** Vary, rewrite, and transform equations to express unknown quantity in terms of known quantities
5. **E – Evaluate** Evaluate expressions, Plug in Numbers, evaluate to determine if answer makes

We will be using this throughout the course.

Group Problem Solving: This also works for group problems solving. The same reason the SOLVE 5-Steps help individuals wrap their heads around a problem, it helps communicate your reasoning to others especially your teacher!

Tolerating Frustration: Quantitative reasoning can be frustrating. A large number of steps and if you make a mistake on any one step you will get a wrong result. That's why we need to tolerate frustration to succeed in quantitative disciplines.

TEAMWORK:

This is very important to employers and is a skill lacking in graduates.

Making unemotional decisions: The largest issue is we are human and emotions get in our way. Here are a few things to assist us in preventing emotions from disrupting teams.

1. Allow everybody enough time to state their opinions without interruption. However we should be clear, complete, and concise in expressing ourselves (see Item 5).
2. Eliminate religion, politics, cursing and racial, ethnic, sexist, or other slurs, offensiveness, and insensitivity. All of these things inflame emotions in people and distract from focusing on the scientific or technical problem you're trying to solve.
3. Avoid personal attacks. Don't say, "your idea stinks." Say something like, "I'm not fond of this idea because …"
4. Explain your reasoning leading your conclusion. DON'T BELIEVE YOU ALREADY KNOW HOW TO EXPLAIN YOURSELF! It takes many years of practice a first or second year college student is poor at explaining their reasoning. I do not consider myself a master. We tend to make "leaps of logic" that even we are not aware of. Get a friend or mentor to listen to you, asking questions when we have take "leaps of logic," and in a non-threatening way help us understand where we need to "fill in the holes" (break our reasoning down into smaller steps). Don't be offended if somebody asks you to explain in greater detail. For your information "leaps of logic" is called a *non sequitur*.
5. Be clear and complete, yet concise and stick to the point. DON'T BELIEVE YOU ALREADY KNOW THIS! Again, a friend or mentor can assist with this. It takes many years of practice to whittle your speech and writing to be concise, yet clear and

sufficient and I don't consider myself a master. Don't be offended if somebody stops you from rambling. Chalk it up to practicing speaking and writing clearly, completely, and concisely.

6. Avoid "My Way or the Highway" thinking, the "Pet Theory," and hyper-criticism. We become upset if one person dominates a discussion, if our ideas are unfairly "shot down," or if somebody is being hypercritical. We're going to discuss brainstorming later as it's an important technique to prevent emotional attachment or rejection of ideas.
7. Participate, but don't dominate. One method is for the facilitator to announce to the group to be clear, complete, and concise in their answers and then poll each member's opinion on an issue.
8. Take a vote and, if necessary, a secret ballot. Resort to this if there is a point of contention that cannot be resolved by consensus and secret ballot if the discussion becomes heated.
9. Take a break. Sometimes giving people some alone cooling off time, adjourning for lunch or the evening, getting a meal together as a team, relaxing, loosening up (imbibe alcoholic beverages or get a massage for those that don't imbibe), go over the ground rules (no personal attacks, stick to the issue, nobody dominates, everybody participates, etc), then resume discussion.
10. We need to respect others opinions and interact civilly in order to WORK together.

Brainstorming: Gather all potential ideas WITHOUT PREJUDGEMENT! Write down even bad ideas. Don't write whose idea it is. Bad ideas are OK – you want everybody comfortable sharing ideas. Let people just shout out an idea. Or have people write it on a post it note and have stick in on a board. Don't say anything about the merit of an idea.

Group Decision Making: The next phase is considering the merit of an idea. Have people discuss the pros and cons. Make a T-chart or use a figure-of-merit (FOM – mathematical formula where the idea that best solves the problem gains the highest or lowest score).

A simple example of an FOM is cost to run a car. Let's say the price of one car is P=$10,000 and another P=$20,000, vehicles are in your fleet t=5 years, resale value is 20% of purchase price (r=0.2), they're used k=48,000 km/yr (30,000 mi), the $10,000 car gets e=8.5 km/l (20 mpg) while the $20,000 care gets e=12.75 km/l (30 mpg) at a future projected average of f=1.3 $/l, and maintenance is m=3 times fuel cost. An FOM would be cost to run the car per year which we are trying to minimize (lowest score).

$$FOM = \frac{P(1-r)}{t} + \frac{kf(1+m)}{e}$$

Crunching the numbers we see that the $10,000 car has an FOM of 31,000 the $20,000 car has an FOM of 23,000. Since FOM is cost per year we see the clear choice is the vehicle with the more expensive price tag. My father was fleet sales manager for a tire company (selling to truckers) and he used this argument to show why his more expensive tires were actually the better bargain. Following is an Excel spreadsheet to calculate this problem:

	A	B	C	D	E	F	G	H	I
1	P	t	r	k	e	f	m	FOM	5 yr cost
2	10,000	5	0.2	48,000	8.5	1.3	3	30,965	154,824
3	20,000	5	0.2	48,000	12.75	1.3	3	22,776	113,882

Most of the time, when we are systematic and mathematically accurate, the choice is clear. And all team members can support the conclusion, believe in the goal, and work hard to accomplish goals. At the end we don't want any grousing about a perceived wrong decision causing poor team morale.

Miscellaneous information on being a good team member:

1. Be on time!
2. 100% attendance (except for REAL hardship like illness, death in family, etc.)!
3. Work while you're at work – don't goof off.
4. Meet commitments consistently and with quality. If you agree to do something by a certain deadline, be 100% reliable that it will be complete and of good quality.

   The last thing an employer wants is somebody who gets assignments in just before the deadline and then the quality is so poor that significant extra work is required. Barely scraping by isn't good enough. Employers want dependable employees – who get things turned in ahead of time and with good quality (little or no revision required). If you're that person and you "get along," you'll be the person selected for promotion.

   Under these circumstances, where you put in a lot of overtime and extra effort, it's reasonable to ask for a few extra days of R&R. Armed forces (probably the people who understand human factors the best) award discretionary R&R passes. If your employer does not, it might be time to get a new employer.

   If your employer makes unreasonable demands and deadlines, it might be time to get a new employer. You may state that you believe a deadline is impossible and that you DO NOT commit to it. Be nice about it and, if overruled, hustle to get it done anyway while polishing up your resume.

   You don't want to get "burned out." Keep the Tarzan philosophy in mind – grab the next vine before you let go of the last one. Your achievement will be of value to another employer and you may even get a promotion.
5. Prepare – in school lab groups, study the lab ahead of time, understand what the lab is about, and write a plan so you don't waste time during the lab. If you can't finish a lab because it took an hour to prepare that's just tough. In the real world you often have time scheduled for a certain piece of equipment, so you've got to use that time wisely. In the real world you also have to keep up with developments in your field – you have to learn new things while holding down your job.
6. In a team divide up the work evenly. In a school lab group rotate duties.

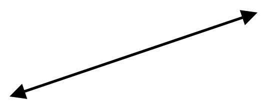
University Physics Workbook Part I
Appendix H: Geometry, Trigonometry, & Vector Summary

Dr. Tovar at Eastern Oregon University has developed a number of good reviews and links to other reviews. If you don't like mine, try his or Dr. Dawkings' at http://physics.eou.edu/opensource/math/, basic algebra at http://physics.eou.edu/opensource/math/alg/alg.pdf, trigonometry at http://physics.eou.edu/opensource/math/trig/trig.pdf, and vectors at http://physics.eou.edu/opensource/math/vecalg/vecalg.pdf. Dr. Dawking's material is at http://tutorial.math.lamar.edu/. Dr. Tovar's and Dr. Dawkings' material are mirrored at my weebly site (http://funphysicist.weebly.com/#helps-mirrored-from-offsite).

The three topics, Geometry, Trigonometry, and Vectors, are closely connected. The summary presented is not exhaustive, however it is my opinion that these are the most relevant to a non-calculus physics classes.

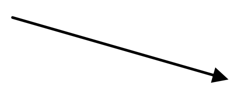
There is some really basic stuff that should not be necessary to review. I presume you already know this material and, therefore, I'll only discuss it briefly.

Point: An object located in space with no width, height, or depth.
Line: A line extends infinitely in both directions and is the set of points such that, if any two points in a line are chosen, all points on the line between those two point are on the shortest path between those two points. A line is pictured as follows:



Note the two arrows at each end. This is an attempt to communicate the idea that a line goes infinitely in both directions. Closely related to the concept of a line are:

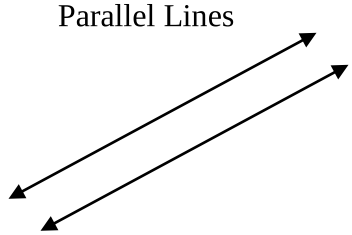
- **Ray:** It is part of a line that starts at one point and continues infinitely. To show this ideas the symbol for a ray is:



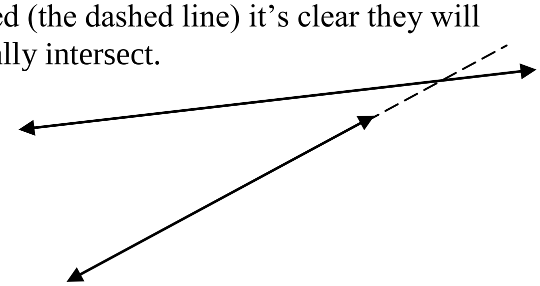
- **Line segment:** This is the part of the line that starts at one point and stops at a second point. To show this idea a line without arrows is drawn:

- **Parallel Lines:** Lines that never intersect and by "intersecting" we mean that at least one point is common to both lines. If lines are not parallel, they will intersect. Examples:

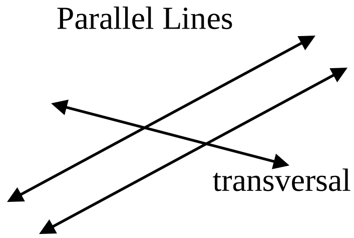
Parallel Lines

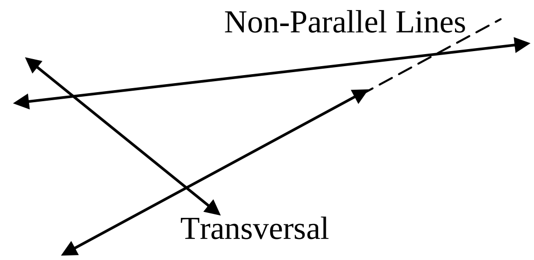


Non-Parallel Lines – note that they are not shown explicitly intersection, however if one or both are extended (the dashed line) it's clear they will eventually intersect.

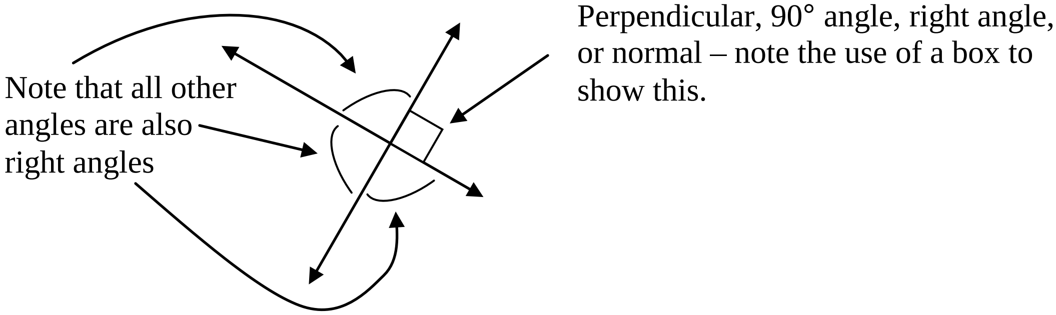


- **Transversal:** A third line that intersects two other lines.





- **Perpendicular:** A line that intersects at 90°, a.k.a., *right angle* or *normal*, to a second line. Note the use of the word "normal." In physics, "normal" has the same meaning as perpendicular. A more rigorous mathematical definition is required, however we'll leave this for later when we define angles. Following are symbols used to show right angles – note the square used to imply this meaning.

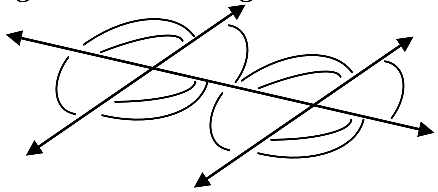


Finally note the use of arcs to denote general angles.

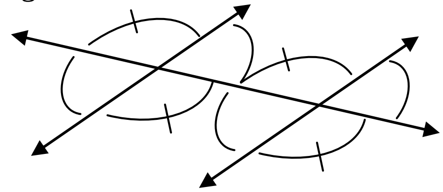
<u>Parallel Lines Transversal</u> – small angles equal each other and large angles equal each other.

The key idea to remember in a transversal of parallel lines is small angles equal each other AND big angles equal each other. Angles are labeled using arcs, arcs with hash marks, a symbol, or some combination of methods. Commonly used symbols are Greek letters like θ or ϕ. Whatever method you use, CLARITY IS MOST IMPORTANT! When working on a geometry problem or physics problem using geometry, USE A RULER & FILL THE ENTIRE SHEET OF PAPER. Here are three different examples of the different notations.

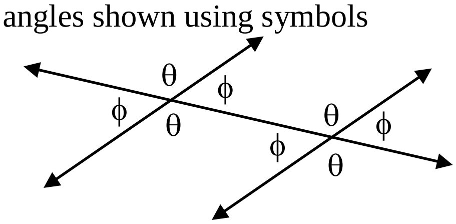
Transversal of two parallel lines with angles shown as single or double arcs



Transversal of two parallel lines with angles shown as arcs or hashed arcs



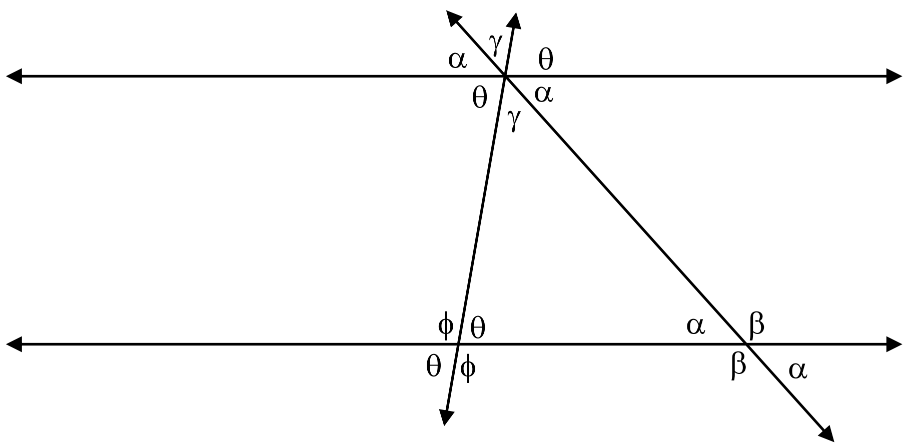
Transversal of two parallel lines with angles shown using symbols



Note that angles ϕ and θ must sum to 180°. Therefore if one angle is known in a transversal of parallel lines, you know all of them. For example, if θ is 120° then ϕ must be 60°.

Angles in Triangle Sum to 180°

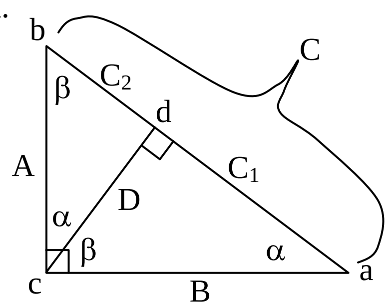
This is actually quite easy to prove using two transversals of parallel lines. Draw the two transversals making sure they both cross the one parallel line at the same point. Then start labeling the angles using the rules of transversals. When we discussed transversals, the angles across from each other have equal measure. We use this fact to assign the symbol γ to the angles near the top. This is shown next:



We finish by noting, viewing the top line, that $\alpha + \gamma + \theta = 180°$, however these are also the interior angles of the triangle formed. Q.E.D. (Latin for "quod erat demonstrandum" meaning "what was required to be proved")

Similar Triangles & Theorem of Pythagoras

There are many proofs of this theorem – I'll present the most obvious proof. We start with a right triangle, that is, a triangle with one right angle and refer to the following diagram.



We label the vertices of the triangle, that is, points where the sides of the triangle intersect, using lowercase letters, the sides of the triangle using uppercase letters, and angles using Greek letters. Note that the vertex opposite the side uses the same letter and the angle at the vertex uses the corresponding Greek letter. Then we drop a perpendicular to the hypotenuse (longest side, line segment ab) and intersecting Point c. Note that "D" refers to the length of the line segment from Point c to Point d. Also note that "C" is the length of the hypotenuse, C_1 is the length from Point d to Point a, C_2 the length from Point d to Point b, and therefore $C = C_1 + C_2$.

Now, to proceed, we must discuss *Similar Triangles*. Any similar figure, including triangles, has the same shape as another, but a different size. For triangles this means angles are preserved, that is, the angles are the same in similar triangles.

Using what we've reviewed previously, that is, the sum of all angles in a triangle equals 180°, we assign symbols to all the angles in the previous figure. After doing this, therefore: Δabc is similar to Δbcd and is similar to Δacd.

Also *in similar figures, all lengths scale the same*. This is the meaning of "same shape, different size." This also means that lengths are in proportion in similar figures and, now, we can write a mathematical equation, a proportion, to describe this:

$\frac{A}{C} = \frac{D}{B} = \frac{C_2}{A}$ and $\frac{B}{C} = \frac{D}{A} = \frac{C_1}{B}$

We will use these equations in just a little bit to derive a second proof of the Pythagorean Theorem. Before we do that, note:

"area of Δbcd" + "area of Δacd" = "area of Δabc."

And areas are proportional to the length of one side squared, therefore the *Pythagorean Theorem* is immediately proven:

$A^2 + B^2 = C^2$

Let's now revisit the equations above applying cross-multiplication:

$A^2 = C * C_2$ and $B^2 = C * C_1$

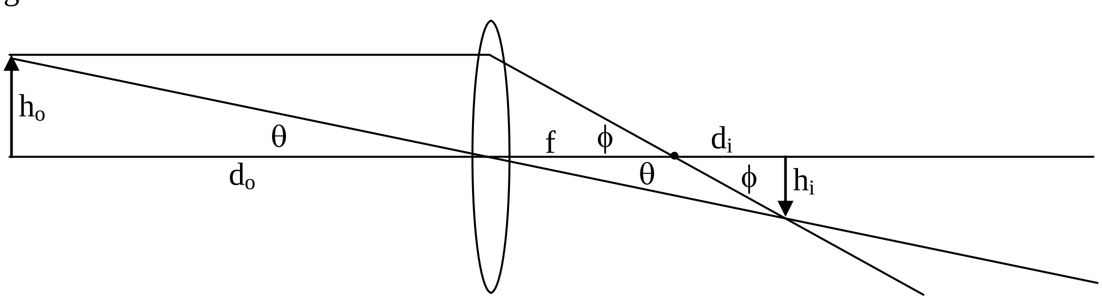
Noting that $C_1 + C_2 = C$ we add the prior equations again deriving the Pythagorean Theorem:

$A^2 + B^2 = C^2$

The Pythagorean Theorem is one of the most important in math and science and we will have opportunity to apply it shortly.

Geometric Optics Application

As the name implies, analysis of optical systems in entirely based on Geometry including similar triangles. There are two basic rules: (1) Rays of light through the center of a lens is undeflected, and (2) parallel rays meet as the focus. Let's draw the figure:



h_o is the size of the object, d_o is the distance of the object from the lens, h_i is the size of the image, d_i is the distance of the image from the lens, f is the focal length of the lens, and angles are labeled to help visualize the similar triangles. Now we can write equations:

$$\frac{d_i}{d_o} = \frac{h_i}{h_o} = M = \text{magnification}$$

And:

$$\frac{h_i}{d_i - f} = \frac{h_o}{f}$$

Rearranging and noting $\frac{d_i}{d_o} = \frac{h_i}{h_o} = M$:

$$\frac{h_i}{h_o} = \frac{d_i}{f} - 1 = \frac{d_i}{d_o}$$

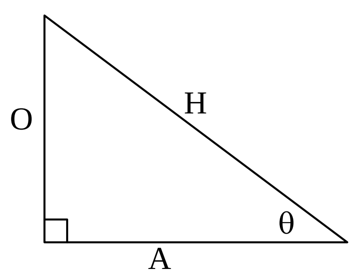
Now by dividing by d_i we obtain the thin lens equation:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

Q.E.D.

Trigonometry – *Some Old Hippy Caught Another Hippy Tripping On Acid*

A humorous saying, but it helps us to remember our trigonometric functions, sine, cosine, and tangent. They are just ratios – refer to the following figure:



O stands for the length of the side opposite from the angle, θ, A stands for the side adjacent to the angle, and H is the hypotenuse of this right triangle. Following is how the trigonometric functions are defined and note, the abbreviation for sine is sin, for cosine is cos, and for tangent is tan:

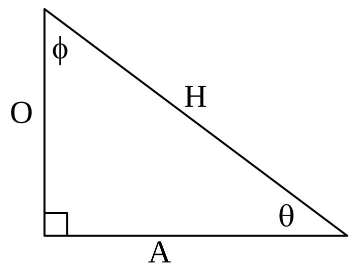
$$\sin(\theta) = \frac{O}{H}$$

$$\cos(\theta) = \frac{A}{H}$$

$$\tan(\theta) = \frac{O}{A}$$

<u>S</u>in equals <u>o</u>pposite over <u>h</u>ypotenuse, <u>c</u>os equals <u>a</u>djacent over <u>h</u>ypotenuse, and <u>t</u>an equals <u>o</u>pposite over <u>a</u>djacent. Now we have a mnemonic device – the underlined letters are there first letters of each word in the phrase: *Some Old Hippy Caught Another Hippy Tripping On Acid*.

Watch out though – take a look at the second non-right angle in the triangle:



The side adjacent to ϕ is NOT the same side that is adjacent to θ. Note that $\phi = 90°\text{--}\ \theta$ and we observe the following relationships:

$$\sin(\phi) = \cos(\theta) = \cos(90^{o} - \phi)$$

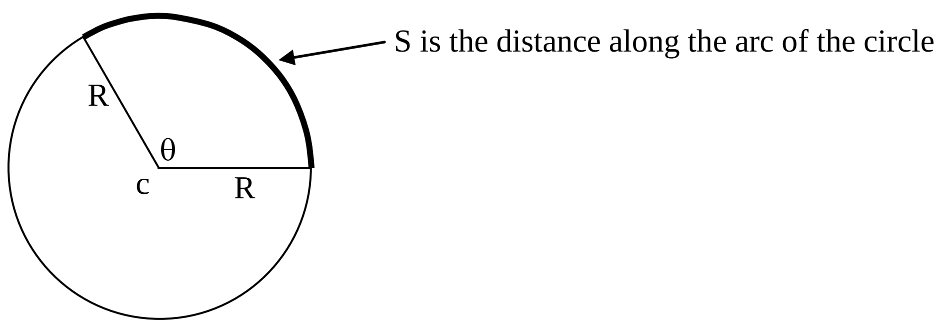
$$\cos(\phi) = \sin(\theta) = \sin(90° - \phi)$$

$$\tan(\phi) = \frac{1}{\tan(\theta)} = \frac{1}{\tan(90° - \phi)} = \cot(90° - \phi)$$

Note the last equation defines the cotangent function.

If we know the angle, we can calculate the ratios, that is, the sin, cos, or tan of the angle. If we know the angle we can calculate the angle, however insure your calculator is in the proper mode. If you wish to know degrees, insure the calculator is in "degree mode" and if you wish to know radians insure your calculator is in "radian mode."

We've been using angles, but we have not been rigorous about defining angle including units of radians, degrees, and revolutions. We'll digress to do this and then discuss inverse trigonometric functions. Refer to the following figure:



A circle is a set of points a specific distance, R, from a center point, c. Shown are two lines from the center to the circle. S is the distance along the circle from the points where the two lines intersect. The angle, in units of radians, is defined as:

$$\theta = \frac{S}{R}$$

If we go all the way around the circle, the distance traveled is $2\pi R$ and thus the angle going in a complete circle is 2π rads (rad is an abbreviation for radian). Going in a complete circle is also called a revolution (rev) and also equals 360°. Now we have conversions between these three angular units.

$$2\pi \text{ rads} = 360° = 1 \text{ rev}$$

Now back to inverse trigonometric functions. Make sure your calculator is in the mode your desire – degrees or radians. I prefer the notation asin, acos, and atan to refer to the inverse functions, however modern calculators use $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$. Many students become confused erroneously thinking the inverse functions refers to a reciprocal and that's why I prefer asin, acos, and atan. However both usages are shown next in defining inverse trigonometric functions.

$$\theta = \text{asin}\left(\frac{O}{H}\right) = \sin^{-1}\left(\frac{O}{H}\right)$$

$$\theta = \text{acos}\left(\frac{A}{H}\right) = \cos^{-1}\left(\frac{A}{H}\right)$$

$$\theta = \text{atan}\left(\frac{O}{A}\right) = \tan^{-1}\left(\frac{O}{A}\right)$$

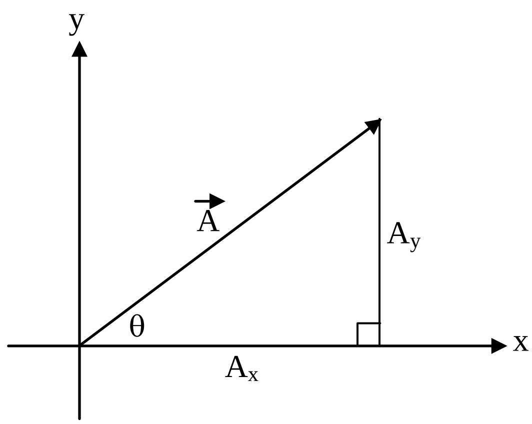
This is all the trigonometry this course requires, however we'll do a trigonometry example in conjunction with vectors, thus we'll discuss vectors first.

Vectors

Nature is not one dimensional – it's at least three or four, including time, or more if the String Hypothesis is confirmed. That's why we need vectors. Before getting much further, let me refer you to an online resource: Online Vector Resource: http://comp.uark.edu/~jgeabana/java/VectorCalc.html.

Vectors will be covered in detail by your instructor and, therefore, this is not a review as previous material was. Hopefully this section can serve as a reference to help you.

We represent vectors by arrows and describe the length, units, and direction of the arrow. If the unit is a Newton (if we're discussing a force), then there is a scale to our arrow, for example, 1 cm length represents 1 N. The direction of the arrow is given by



the angle from the x-axis with counter-clockwise being positive angles. Consider the following example:

A vector is symbolized by an arrow over the letter, such as shown above, a line over the letter, e.g., $\bar{A}$, or bold, **A**. Part of the reason we do things is convenience of the tools available, therefore we use bold letters to symbolize vectors because it was easier for typewriters and that practice continued into our computer age. A line over a letter works well for blackboards. We have better word processing tools these days and thus I'll avoid bold face for vectors, but not all the way to an arrow over to denote vectors. My practice will be the middle – I'll use a line over.

A_x is the x component of the vector and A_y is the y component. There may also be a z component, A_z. It's easy to see, using the Pythagorean Theorem, that the length or *magnitude* of $\overline{A}$ equals:

$$|\overline{A}| = A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Some people use the ordinary capital letter without bold to mean the vector length. I prefer using the absolute value symbol with the overline, that is, the magnitude of $\overline{A}$ is symbolized by $|\overline{A}|$. Also the term "magnitude" is preferred over "length." If we restrict our consideration to two dimensions, the angle, θ, is given by:

$$\theta = \text{atan}\left(\frac{A_y}{A_x}\right)$$

Conversely, in two dimensions, if you know magnitude and angle, you can find the components:

$$A_x = |\overline{A}| \cos(\theta)$$

$$A_y = |\overline{A}| \sin(\theta)$$

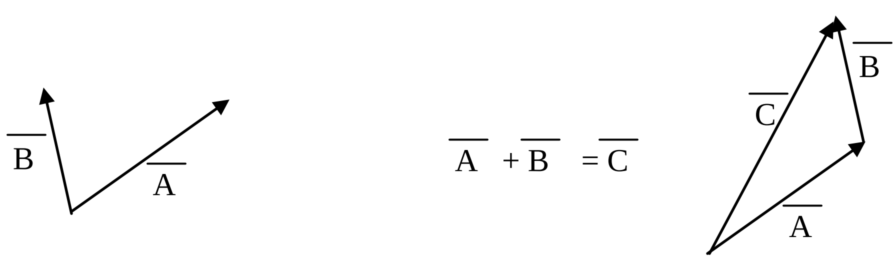
These formulae for 3 dimensions are more complicated and are usually discussed in more advanced classes.

A very popular method to describe vectors is to use the unit vector notation. A unit vector along the x axis, y axis, or z axis is defined. There are just vectors with magnitude of one. The x axis unit vector has a carrot or hat over the letter, that is, $\hat{x}$, y axis unit vector is $\hat{y}$, and z axis is $\hat{z}$. Some authors use i, j, and k instead of x, y, and z to refer to unit vectors. Therefore, $\hat{x} = \hat{\imath}$, $\hat{y} = \hat{\jmath}$, and $\hat{z} = \hat{k}$. The connection to components is as follows:

$$\overline{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$$

Adding Vectors

Graphically we use the arrow description and add vectors tip-to-tail, that is, the tip of the first vector touches the tail of the second. The following figure shows this:



You may use a ruler and protractor to carefully measure and draw to find $\overline{C}$. The component method is extremely popular for this purpose since:

$$C_x = A_x + B_x$$

And

$$C_y = A_y + B_y$$

And, in three dimensions

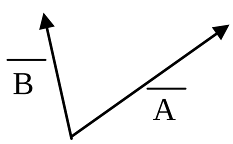
$$C_z = A_z + B_z$$

Or using unit vectors as we will use in the rest of this book

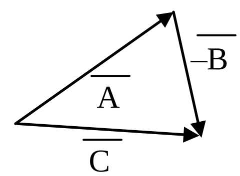
$$\overline{C} = \overline{A} + \overline{B} = (A_x + B_x)\hat{x} + (A_y + B_y)\hat{y} + (A_z + B_z)\hat{z}$$

Subtracting Vectors

This easiest way to think about this is adding the negative vector, that is, a vector the same magnitude and in the opposite direction. So:



$\overline{A} - \overline{B} = \overline{C}$



The way to negate a vector is simply to negate each of the components. This should be clear from the previous figure. So subtraction using components is also quite simple.

$$\overline{C} = \overline{A} - \overline{B} = (A_x - B_x)\hat{x} + (A_y - B_y)\hat{y} + (A_z - B_z)\hat{z}$$

Multiplying Vectors by a Scalar

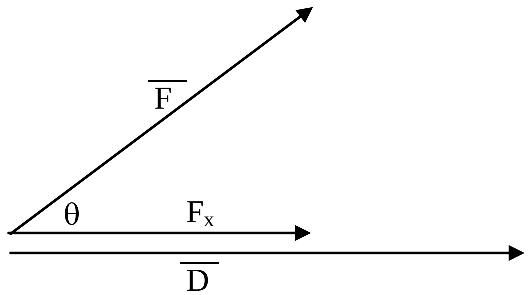
A *scalar*, unlike a vector, is a quantity without direction. Scalars are the stuff we've been used to working with in most of our math. To multiply a vector by a scalar we just multiply the magnitude by the scalar and *direction does not change*. If the scalar is negative, the arrows end flips just like we saw previously when subtracting vectors. Letting the scalar be symbolized by k, then:

$$\overline{C} = k\overline{A} = kA_x\hat{x} + kA_y\hat{y} + kA_z\hat{z}$$

Vector Scalar Product aka Dot Product

We first encounter this when discussing work, that is, the force in the direction of motion times distance. If the force is perpendicular to the direction of motion, there is no work. It's like the weight of a rider on a riding lawn mower. Motion is parallel to the ground, but the person's force is perpendicular to the ground, therefore the person isn't working. They may be sipping an ice cold lemonade lazily guiding the lawn mover. The lawn mover's motor is doing the work – not the rider.

We say that work is force in the direction of motion – a quantity we can easily derive as follows:



F_x is the force in the direction of the motion, $\overline{D}$, and since $F_x = |\overline{F}|\cos(\theta)$:

$$\overline{F} \cdot \overline{D} = |\overline{F}||\overline{D}|\cos(\theta)$$

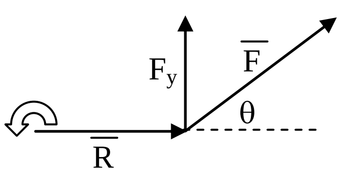
This is the definition of the dot product (note the "dot" between vectors) or scalar product. The result is a scalar.

The proof of the component version of the dot product is beyond this course and is simply stated below:

$$\overline{F} \cdot \overline{D} = F_x * D_x + F_y * D_y + F_z * D_z$$

Vector Cross Product

We first encounter this discussing torque and rotational motion. Torque is defined as force *perpendicular to the lever arm*, R, times the lever arm. See the next figure:



In this case it's clear that only the *y* component of force will cause rotation. F_x will not. Therefore the magnitude of torque is easy to calculate and is:

$$|\overline{\tau}| = |\overline{R}||\overline{F}|\sin(\theta)$$

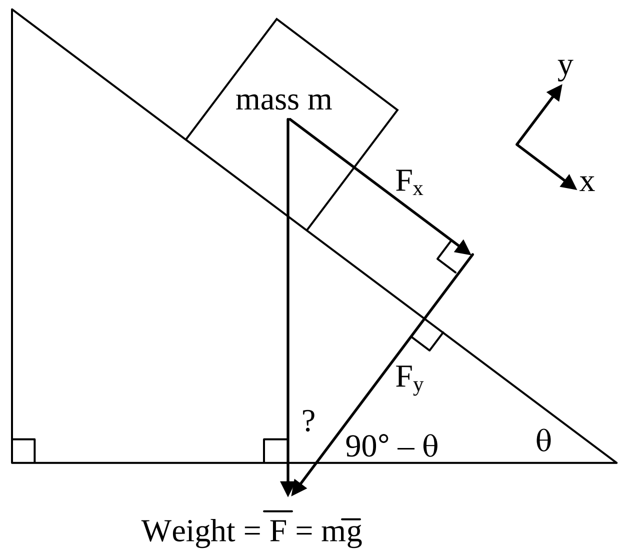
Torque is a vector quantity and one finds the direction of the vector using the "Right Hand Rule." Point the thumb of your right hand in the direction of $\overline{R}$, and fingers in the direction of the force component perpendicular to $\overline{R}$, then your palm will point in the direction of torque. This takes practice and will be covered in class.

The component version of the cross product is beyond the scope of this course and is simply stated below:

$$|\overline{\tau}| = \overline{R} \times \overline{F} = (R_y * F_z - R_z * F_y)\hat{x} + (R_z * F_x - R_x * F_z)\hat{y} + (R_x * F_y - R_y * F_x)\hat{z}$$

Application – Force Acting Down Plane

This is a common problem in physics and illustrates the use of Geometry, Trigonometry, and Vectors. Weight acts downward, but can be resolved into two components – a component acting parallel to the plane and a component acting normal (perpendicular) as shown:



First note that we've tilted the x and y axes so the component of force acting parallel to the plane is F_x and the component acting normal is F_y. But we don't know the angle of the force vector. It's easy to figure out, however, by noting that two angles (θ and a right angle) are known in the lower right triangle making the third angle 90° - θ. Therefore the angle labeled ? must be θ.

Using trigonometry, $F_x = m|\overline{g}|\sin(\theta)$ and $F_y = -m|\overline{g}|\cos(\theta)$. This is counterintuitive based on our vector discussion. This highlights the need to carefully think through a problem and don't just rely on formulae.

The issue is that the angle of the force vector from the x axis is not θ, it is negative (angle is going clockwise) 90° - θ. Therefore:

$$F_x = m|\overline{g}|\cos(\theta - 90°) = m|\overline{g}|\sin(\theta)$$

And

$$F_y = m|\overline{g}|\sin(\theta - 90°) = -m|\overline{g}|\cos(\theta)$$

Conclusion

We've summarized Geometry, Trigonometry, and Vectors as they related to Algebra based physics classes. Although we've summarized key material, applying this takes practice. Also be careful in analyzing problems. Simply relying on formulae is insufficient – you must think through problems, apply appropriate formulae, know how to correctly apply those formulae, combine knowledge with other knowledge, and do your math carefully step-by-step.

Appendix I: Exponentials & Logarithms Summary

In the prior appendix I mentioned Dr. Dawkings has developed a number of good reviews at http://tutorial.math.lamar.edu/ and Dr. Tovar has other reviews and links to Dr. Dawkings' reviews at http://physics.eou.edu/opensource/math/. In particular, Dr. Dawkings' algebra and trigonometry review contains exponentials and logarithms at http://tutorial.math.lamar.edu/pdf/AlgebraTrig.pdf. Dr. Tovar's and Dr. Dawkings' material are mirrored at my weebly site (http://funphysicist.weebly.com/#helps-mirrored-from-offsite).

Laws of Exponents

To remember laws of exponents, you may expand recalling that exponents mean repeated multiplication. For example, $x^3 = x*x*x$. All of the rules are derived from this basic understanding.

Let i, j, m and n be any Real or Rational (fraction) numbers. Let a and b be any Real or Rational numbers greater than zero. When first learning laws of exponents think of i, j, m, n, a and b as integers, however they are quite general and are extended to complex numbers.

For the radical form recall: $\sqrt[n]{a^m} = (a)^{\left(\frac{m}{n}\right)}$ and $\sqrt{a} = (a)^{\left(\frac{1}{2}\right)}$

Rule Name	Summary	Radical Form	Examples
Zero Exponent	$a^0 = 1$	not applicable	$123^0 = 1$
Exponent of One	$a^1 = a$	not applicable	$123^1 = 123$
Exponents and Roots	$\left(a^{\left(\frac{1}{n}\right)}\right)^n = a$ $(a^n)^{\left(\frac{1}{n}\right)} = a$	$\sqrt[n]{a^n} = \left(\sqrt[n]{a}\right)^a = a$	$\left(\sqrt[3]{123}\right)^3 = 123$ $\sqrt{3^2} = 3$
Negative Exponent	$a^{-n} = \frac{1}{a^n}$	$\sqrt[-n]{a} = \frac{1}{\sqrt[n]{a}} = \sqrt[n]{\frac{1}{a}}$	$4^{-1} = \frac{1}{4}$ $\sqrt[-3]{2} = \frac{1}{\sqrt[3]{2}}$ $\sqrt[-3]{2} = \sqrt[3]{\frac{1}{2}}$
Powers of One	$1^n = 1$	$\sqrt[n]{1} = 1$	$1^{456} = 1$ $\sqrt[456]{1} = 1$

Rule Name	Summary	Radical Form	Examples
Powers of -1	n even: $(-1)^n = 1$ n odd: $(-1)^n = -1$	n even: $\sqrt[m]{(-1)^n} = 1$ n & m odd: $\sqrt[m]{(-1)^n} = -1$ n odd & m even results in imaginary & complex numbers (don't worry about this yet)	$(-1)^{456} = 1$ $(-1)^{457} = 1$ $\sqrt[3]{-1} = \sqrt[457]{-1}$ $\sqrt[3]{-1} = -1$ $\sqrt[4]{(-1)^2} = -1$ $\sqrt[3]{(-1)^2} = -1$ $\sqrt{-1} = i$ (defines pure imaginary number of magnitude 1)
Product of Powers	$a^i a^j = a^{(i+j)}$	Note: $\sqrt[n]{a^m} = (a)^{\left(\frac{m}{n}\right)}$ & $\sqrt[q]{a^p} = (a)^{\left(\frac{p}{q}\right)}$ Thus: $\sqrt[n]{a^m}\ \sqrt[q]{a^p} = (a)^{\left(\frac{m}{n}+\frac{p}{q}\right)}$	$2^3 * 2^5 = 2^{(3+5)}$ $2^3 * 2^5 = 256$
Quotient of Powers	$\frac{a^i}{a^j} = a^{(i-j)}$	See Product of Powers: $\frac{\sqrt[n]{a^m}}{\sqrt[q]{a^p}} = a^{\left(\frac{m}{n}-\frac{p}{q}\right)}$	$\frac{6^7}{6^5} = 6^{(7-5)}$ $\frac{6^7}{6^5} = 36$ $\frac{\sqrt{6^7}}{\sqrt{6^5}} = 6^{\left(\frac{7}{2}-\frac{5}{2}\right)}$ $\frac{\sqrt{6^7}}{\sqrt{6^5}} = 6$
Power of Power	$(a^m)^n = a^{m*n}$	$\sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$ $\sqrt[n]{a^m} = a^{\left(\frac{m}{n}\right)}$	$(2^3)^{-2} = 2^{-6}$ $(2^3)^{-2} = \frac{1}{64}$ $\left(\sqrt[3]{5}\right)^6 = 5^2 = 25$
Power of Product	$(a * b)^m = a^m * b^m$	$\sqrt[n]{(a * b)^m} = \sqrt[n]{a^m} * \sqrt[n]{b^m}$	$(2 * 3)^2 = 6^2$ $(2 * 3)^2 = 36$ $\sqrt{4 * 9} = \sqrt{4} * \sqrt{9}$ $\sqrt{4 * 9} = 6$

Rule Name	Summary	Radical Form	Examples
Power of Quotient	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\sqrt[m]{\frac{a}{b}} = \frac{\sqrt[m]{a}}{\sqrt[m]{b}}$	$\left(\frac{3}{5}\right)^{-2} = \frac{3^{-2}}{5^{-2}}$ $\left(\frac{3}{5}\right)^{-2} = \frac{25}{9}$ $\sqrt{\frac{25}{9}} = \frac{\sqrt{25}}{\sqrt{9}}$ $\sqrt{\frac{25}{9}} = \frac{5}{3}$

Logarithms

Remember, the logarithm function is defined as the inverse function to the exponential, that is: $\log_a(a^x) = x$ where *a* is the base of the logarithm. And, conversely, $a^{\log_a(y)} = y$.

Note that there are two very common bases, 10 and *e*, for logarithms. Base 10 is rather obvious – we're used to dealing in base 10. Often the base is left off if referring to base 10. $\log_{10}(x) = \log(x)$ mean the same thing. Base *e* is not so obvious and it requires a study to calculus to understand why this is an important base. For right now, before your study of calculus, you will need to accept that *e* is important and learn to work with natural logs, that is, logs of base *e*. Also a common notation for logs of base *e* is ln(x) and *e* is an irrational number ≈ 2.71828.

Since logarithms are the inverse function of exponentials, for every law of exponentials above, there is a corresponding law of logarithms. We repeat the first two columns of the above table, the third column is the corresponding law for logarithms, and the last column shows examples.

Rule Name	Law of Exponents	Law of Logarithms	Examples
Inverse Function	$a^{\log_a(y)} = y$	$\log_a(a^x) = x$	$\log(10^3) = 3$ $\ln(e^2) = 2$ $\log_2(64) = 6$
Zero Exponent	$a^0 = 1$	$\log_a(1) = 0$	$\log(1) = 0$ $\ln(1) = 0$ $\log_2(1) = 0$
Exponent of One	$a^1 = a$	$\log_a(a) = 1$	$\log(10) = 1$ $\ln(e) = 1$ $\log_2(2) = 1$
Power of Power	$(a^m)^n = a^{m*n}$	$\log_a((a^m)^n) = m*n$	$\log((1000)^2) = 6$ $\ln((e^4)^5) = 20$ $\log_2((8)^3) = 9$

Rule Name	**Law of Exponents**	**Law of Logarithms**	**Examples**
Change Base	Use Power of Power Rule and fact that $a^{\log_a(b)} = b$ therefore (aka ∴) $b^x = a^{x*\log_a(b)}$	$\log_a(b^x) = x*\log_a(b)$	$\log(8) = 3*\log(2)$ $\log_2(100) = 2*\log_2(10)$ $\ln\left(\left(\frac{1}{2}\right)^{\left(\frac{t}{t_h}\right)}\right) = \frac{t}{t_h}*\ln\left(\frac{1}{2}\right)$ If $\frac{V}{V_o} = e^{\left(-\frac{t}{RC}\right)}$ then $\ln\left(\frac{V}{V_o}\right) = -\frac{t}{RC}$
Logs of Different Bases	Use Change Base Rule and let $x = \log_b(y)$ ∴ $\log_a(y) = \log_b(y)*\log_a(b)$ And ∴ $\frac{\log_a(y)}{\log_a(b)} = \log_b(y)$ Let y = a ∴ $\frac{1}{\log_a(b)} = \log_b(a)$		$\log_2(14) = \frac{\ln(14)}{\ln 2}$ $\log_2(14) = \frac{\log(14)}{\log(2)}$ $\ln(14) = \frac{\log(14)}{\log(e)}$ $\ln(14) = \ln(10)*\log(14)$ $\ln(14) \approx 2.303*\log(14)$
Negative Exponent	$a^{-n} = \frac{1}{a^n}$	$\log_a\left(\frac{1}{a^n}\right) = -n$ $\log_a(a^{-n}) = -n$	$\ln\left(\frac{1}{e}\right) = -1$ $\log(10^{-19}) = -19$
Powers of -1	n even: $(-1)^n = 1$ n odd: $(-1)^n = -1$	The logarithm of a negative number is undefined	
Product of Powers	This Rule, $a^i a^j = a^{(i+j)}$, implies $\log_a(x*y) = \log_a(x) + \log_a(y)$		$\log(2*10^4) = 4*\log(2)$
Quotient of Powers	This Rule, $\frac{a^i}{a^j} = a^{(i-j)}$, implies $\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$		$\log\left(\frac{14}{1000}\right) = \log(14) - 3$
Power of Product	$(a*b)^m = a^m*b^m$	While the law of exponents is simple and important rules, there is no simple corresponding logarithm rule. Apply previous rules in multi-step analysis. Examples: $\log((3*100)^2) = 2*\log(3) + 2*2$ $\log\left(\left(\frac{3}{100}\right)^2\right) = 2*\log(3) - 2*2$	
Power of Quotient	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$		

University Physics Workbook Part I
Appendix J: Calculus Review

Fortunately sparkcharts makes this available for free at http://www.sparknotes.com/free-pdfs/nook-study/9781411487628_calculusi.pdf. It also has a pretty good Logarithm, Exponent, and Trigonometry review. It does not include a review of Geometry and Vectors that I include in Appendic D. By copyright law I can't reproduce that sparkchart here. However Appendices D & E include Trignometry, Exponentials, and Logarithms and following is a "Quick Calculus" review by Dr. Tovar of Eastern Oregon University.

Dr. Tovar created several good reviews including calculus (http://physics.eou.edu/opensource/math/calc/calc.pdf). He does permit reproduction which I will do imbedding his review into this workbook. Note that the very last line on each page is not Dr. Tovar's – it is this workbook's footer. Following in Dr. Tovar's calculus review starting with the title page below:

Differential and Integral Calculus Review and Tutorial

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La Grande, Oregon, 97850

January 28, 2009

Tutorial 1

Calculus Review

Contents

This tutorial is a review of the basic results of differentiation and integration. Of course some of the results may be new to some of the readers. Hopefully, those readers will find the new results interesting as well as informative.

There are many things one could say about the history of calculus, but one of the most interesting is that integral calculus was first developed by Archimedes of Syracuse OVER 2250 YEARS AGO! He was a very interesting guy. You can google him to learn more, but I highly recommend the (historical fiction) book "The Sand Reckoner" by Gillian Bradshaw which is a story of his life.

1.1 What are Elementary Functions?

The Elementary functions are:

1. Polynomials (of integer and complex order)
2. Exponential and Logarmithmic Functions
3. Sinusoidal and Inverse Sinusoidal Functions
4. Hyberbolic Sinusoidal and Inverse Hyperbolic Sinusoidal Functions
5. Any finite number of Sums, Products, or Compositions of Elementary Functions

Here are some examples of elementary functions:

Elementary Function	Examples
Polynomials	$a_3x^3 + a_2x^2 + a_1x + a_0$
Exponential and Logarmithmic Functions	e^{ax}, $ln(ax)$
Sinusoidal and Inverse Sinusoidal Functions	$cos(ax)$, $tan^{-1}(ax)$
Hyberbolic Sinusoidal and Inverse Hyperbolic Sinusoidal Functions	$cosh(ax)$, $tanh^{-1}(ax)$
Composite Elementary Function	$\frac{e^{sin(x)+x^2}}{cosh(x)} + ln(7x)$

So, what isn't an elementary function? There are certain integrals and differential equations that "can't be solved" so instead of solving them, we name them. For example, Bessel functions are solutions to "Bessel's Equation." Nonelementary functions that have a special name are known as **Special Functions**. Another common example of a special function is the Error Function which is the solution to the integral

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-\hat{x}^2} d\hat{x} \tag{1.1}$$

1.2 Differential Calculus

Fast Facts:

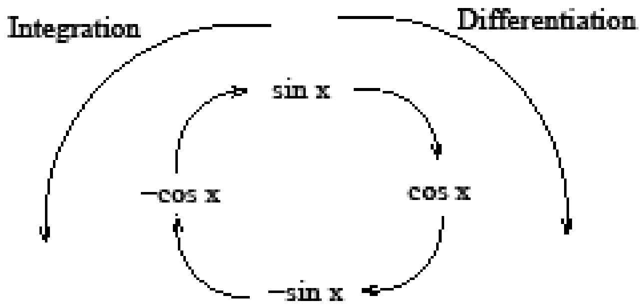
1. Definition: $f'(x) = lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$.
2. Derivative is an operator (it operates on functions).
3. In particular, the derviative is the slope operator. Thus, it represents a "rate of change." When the independent variable is time, the derivative becomes a time rate of change. The time rate of change of position is velocity, the time rate of change of velocity is acceleration, and the time rate of change of acceleration is jerk. It can be readily seen that the units of the time derivative of f are $f/time$.
4. The inverse operator is the antiderivative or integral (This is the Fundamental Theorem of Calculus).
5. The Integral is the Area Operator.
6. The Derivative of any Elementary Function is an Elementary Function.

1.2.1 Some Derivatives

Here is a very short table of derivatives:

Function	Derivative
x^n	nx^{n-1}
e^x	e^x
$ln(x)$	$\frac{1}{x}$
$sin(x)$	$cos(x)$
$cos(x)$	$-sin(x)$
$-sin(x)$	$-cos(x)$
$-cos(x)$	$sin(x)$

One may notice that the derivatives for sin and cos follow a simple pattern:



1.2.2 Basic Theorems

Theorem Name	Theorem
Chain Rule	$\frac{d}{dx}\left(A(B(x)\right) = \frac{dA(B)}{dB}\frac{dB(x)}{dx}$
Linearity	$\frac{d}{dx}\left(aA(x) + bB(x)\right) = a\frac{dA}{dx} + b\frac{dB}{dx}$
Product Rule	$\frac{d}{dx}\left(A(x)B(x)\right) = A(x)\frac{dB}{dx} + \frac{dA}{dx}B(x)$
Quotient Rule	$\frac{d}{dx}\left(\frac{A(x)}{B(x)}\right) = \frac{B(x)\frac{dA}{dx} - A(x)\frac{dB}{dx}}{B^2(x)}$

1.2.3 Implicit Differentiation

Implicit differentiation is used when you do not have an explicit solution for the dependent variable of interest. Here is an example:
Find dy/dx of the following function (it's the equation of a circle):

$$x^2 + y^2 = 1 \tag{1.2}$$

Differentiating each of the terms yields

$$2x + 2y\frac{dy}{dx} = 0 \tag{1.3}$$

or

$$\frac{dy}{dx} = -x/y \tag{1.4}$$

A typical application would be the max/min problem, which could be accomplished by setting $dy/dx = 0$ which yields $x = 0$. This occurs when $y^2 = 1$ or $y = \pm 1$, as expected.

1.2.4 Logarithmic Differentiation

One would like to extend the product rule for more than two functions. This can be achieved with Logarithmic Differentiation. Suppose you want to find the derivative of

$$y(x) = A(x)B(x)C(x)D(x) \tag{1.5}$$

You could apply the product rule many times, or take the logarithm of both sides:

$$ln(y(x)) = ln(A(x)) + ln(B(x)) + ln(C(x)) + ln(D(x)) \tag{1.6}$$

where a simple property of the logarithm has been used. Now, taking the derivative of both sides yields

$$\frac{1}{y(x)}\frac{dy}{dx} = \frac{1}{A(x)}\frac{dA}{dx} + \frac{1}{B(x)}\frac{dB}{dx} + \frac{1}{C(x)}\frac{dC}{dx} + \frac{1}{D(x)}\frac{dD}{dx} \tag{1.7}$$

Finally, multiplying both sides by $y(x)$ yields

$$\frac{dy}{dx} = B(x)C(x)D(x)\frac{dA}{dx} + A(x)C(x)D(x)\frac{dB}{dx} + A(x)B(x)D(x)\frac{dC}{dx} + A(x)B(x)C(x)\frac{dD}{dx} \tag{1.8}$$

Of course, if one is so inclined, one could generalize the results. If $y(x)$ is written

$$y(x) = \prod_{i=1}^{N} A_i(x) \tag{1.9}$$

then

$$\frac{dy}{dx} = \sum_{j=1}^{N} \frac{1}{A_j(x)} \frac{dA_j(x)}{dx} \prod_{i=1}^{N} A_i(x) \tag{1.10}$$

1.2.5 Pascal's Triangle

Consider Pascal's Triangle:

$$\begin{array}{c} 1 \\ 1 \quad 1 \\ 1 \quad 2 \quad 1 \\ 1 \quad 3 \quad 3 \quad 1 \\ 1 \quad 4 \quad 6 \quad 4 \quad 1 \\ 1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1 \\ 1 \quad 6 \quad 15 \quad 20 \quad 15 \quad 6 \quad 1 \end{array}$$

Each of the numbers are obtained by adding the two adjacent numbers in the row above it. For example, 10 is below it's adjacent 4 and 6.

The rows of Pascal's Triangle represent the binomial coefficients, so that

$$(a+b)^3 = \mathbf{1} \cdot a^3 b^0 + \mathbf{3} \cdot a^2 b^1 + \mathbf{3} \cdot a^1 b^2 + \mathbf{1} \cdot a^0 b^3 \tag{1.11}$$

or more simply

$$(a+b)^3 = a^3 + 3a^2 b + 3ab^2 + b^3 \tag{1.12}$$

There is a formula for the binomial coefficients, given the "Choose Function":

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} \tag{1.13}$$

where n is the row and k goes from 1 to n. For example, the third number on the 5th row of Pascal's Triangle is a 10, but

$$\binom{5}{3} = \frac{5!}{3!(5-3)!} = \frac{5 \cdot 4}{2 \cdot 1} = 10 \tag{1.14}$$

Note that the top solitary "1" is considered to be in the zeroth row and that $0! = 1$ by convention.

This relates to differential calculus, because there is a similar relationship that applies to the product rule to multiple derivatives. Suppose we are interested in the 4^{th} derivative of a product:

$$y^{(4)}(x) = (A(x)B(x))'''' \tag{1.15}$$

While the derivative terms that must exist are straightforward, it also happens that the coefficients of the derivatives are given by Pascal's Triangle, so that in this case

$$y^{(4)}(x) = A''''(x)B(x) + 4A'''(x)B'(x) + 6A''(x)B''(x) + 4A'(x)B'''(x) + A(x)B''''(x) \tag{1.16}$$

Thus, by inspection it can be seen that the 4^{th} derivative of xe^x is

$$\frac{d^4}{dx^4}(xe^x) = (4+x)e^x \tag{1.17}$$

as the first 3 terms are zero. By inspection, can you determine the 4^{th} derivative of $x^2 e^x$? (See Bottom of Page for answer)[1]

[1] $(12 + 8x + x^2)e^x$

1.3 Integral Calculus

Fast Facts:

1. Definition of a Definite Integral: $\int_a^b f(x)dx = lim_{N\to\infty}\left(\frac{b-a}{N}\right)\sum_0^{N-1} f\left(a + i\left(\frac{b-a}{N}\right)\right)$.
2. Definition of an Indefinite Integral: $\int f(x)dx = \int_a^x f(\hat{x})d\hat{x} + C$
3. The Indefinite Integral is an operator (it operates on functions).
4. In particular, the Indefinite Integral is the Accumulated Area Operator. The area is achieved by summing many rectangles of length $\Delta x = (b-a)/N$ and height $f(a + i\Delta x)$. Thus, the units of $\int_a^b f(x)dx$ is the units of $f(x)$ multiplied by the units of x.
5. The Indefinite Integral of any Elementary Function **may or may not be** an Elementary Function.

1.3.1 Some Integrals

Function	Simple Form	Advanced Form
$x^n \ \ (n \neq -1)$	$\frac{x^{n+1}}{n+1}$	$\int f^n(x)\frac{df}{dx}dx = \frac{f^{n+1}(x)}{n+1}$
x^{-1}	$ln\|x\|$	$\int \frac{df/dx}{f}dx = ln\|f(x)\|$
e^x	e^x	$\int e^{f(x)}\frac{df}{dx}dx = e^{f(x)}$
$ln(x)$	$xln(x) - x$	$\int ln(f(x))\frac{df}{dx}dx = f(x)ln(f(x)) - f(x)$
$sin(x)$	$-cos(x)$	$\int sin(f(x))\frac{df}{dx}dx = cos(f(x))$
$-cos(x)$	$-sin(x)$	
$-sin(x)$	$cos(x)$	
$cos(x)$	$sin(x)$	

1.3.2 Techniques of Integration

Technique	When to Use
u-Substitution	When it's obvious or when you're stuck.
Integration by Parts	When you have a product of two functions, and you know the derivative of one and the integral of the other.
Trigonometric Substitution	When you have $(a + x^2)$ or $(a - x^2)$ terms.
Synthetic Division/Partial Fraction	When you have a ratio of polynomials.
Series Solution	When you stuck but realize a Taylor Series is easy to calculate.

As you may recall, the formula for integration by parts is

$$\int udv = uv - \int vdu \tag{1.18}$$

A common mistake when using integration by parts on a definite integral is to forget to evaluate the uv term with the integration limits.

1.3.3 Examples of Integration Techniques

Example (Advanced Forms)

$$\int \frac{x^2}{1+x^3}dx = \frac{1}{3}\int \frac{3x^2}{1+x^3}dx = \frac{1}{3}ln|1+x^3| + C \tag{1.19}$$

Comment: The "Advanced Forms" involve what I call the "Hope Method." You see a complicated function and hope that the derivative of the inside of the equation is sitting on top. In this case it is. One is sometimes taught to use u-Substitution here, but this integral should be done in your head. Notice how the integral would be much harder if it is $\int \frac{1}{1+x^2}dx$. That's why it's the Hope method, you hope that derivative is there!

Example (Advanced Forms)

$$\int x^5 ln(x)dx = \frac{1}{6}\int x^5 ln(x^6)dx = \frac{1}{36}\int 6x^5 ln(x^6)dx = \frac{1}{36}x^6 ln|x^6| - x^6 + C \tag{1.20}$$

Comment: One of my favorite integrals. By using the properties of the logarithm, we can make the derivative show up by just multiplying by a constant. The integral could be done by integrating by parts, but it would be longer and much less cool!!

Example (Integration by Parts)

$$\int \underbrace{x}_{u}\underbrace{e^x dx}_{dv} = \underbrace{x}_{u}\underbrace{e^x}_{v} - \int \underbrace{e^x}_{v}\underbrace{dx}_{du} = e^x(x-1) + C \tag{1.21}$$

Comment: You can see that if we had $\int x^n e^x dx$ we would perform integration by parts n times, as the power of the monomial decreases by 1 every time, but the exponential stays firm. It is a good idea to take the derivative of your result to insure a correct answer. In this case the derivative is obtained by the product rule: $e^x(1) + (x-1)e^x = xe^x$ as it must.

Example (Synthetic Division)

$$\int \frac{x^3+2x^2+2x+1}{x+2}dx = \int x^2 + 2 - \frac{3}{x+2}dx = \frac{x^3}{3} + 2x - 3ln|x+2| + C \tag{1.22}$$

Comment: Use synthetic division on a rational function when the degree of the polynomial on top is greater than or equal to that on the bottom. Some students haven't done synthetic division in a while. It's just the same as long division you first learned in grade school. Unfortunately, you may have not done that in a while either!

Example (Partial Fraction Expansion)
The integrand of

$$\int \frac{1}{x^2+6x+8}dx \tag{1.23}$$

can be reduced by using a technique in ALGEBRA known as Partial Fraction Expansion. In particular, the integrand can be rewritten

$$\frac{1}{x^2+6x+8} = \frac{1}{(x+2)(x+4)} = \frac{A}{x+2} + \frac{B}{x+4} \tag{1.24}$$

Solving for A and B yields $A = 1/2$, $B = -1/2$. The solution is

$$\int \frac{1}{x^2+6x+8}dx = \frac{1}{2}[ln|x+2| - ln|x+4|] = \frac{1}{2}ln\left|\frac{x+2}{x+4}\right| + C \tag{1.25}$$

Comment: For the integral of a fifth degree polynomial divided by a second degree polynomial, one would use synthetic division first, and use Partial Fractions on the remaining integral. There is a technique where the partial fractions coefficients can be determined by inspection.

Example (Series Solution)
To solve

$$\int e^{-x^2} dx \tag{1.26}$$

we may note that the Taylor Series expansion for an exponential function is

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \tag{1.27}$$

Thus, the integral becomes

$$\int e^{-x^2} dx = \int \sum_{n=0}^{\infty} \frac{(-x^2)^n}{n!} dx = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \int x^{2n} dx = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \frac{x^{2n+1}}{2n+1} \tag{1.28}$$

Comment: If you multiply our result by $2/\sqrt{\pi}$, we just found the Taylor Series expansion for erf(x).

Example (Trigonometric Substitution)
Find the area of a cirle. The equation of a circle is $x^2 + y^2 = r^2$. To find the area we will double the area of the top half of the circle, which can be found be integration:

$$A = 2 \int_{-r}^{r} \sqrt{r^2 - x^2} dx \tag{1.29}$$

Making the substitution $x = rcos(\phi)$, it follows that $dx = -rsin(\phi)d\phi$, and that

$$A = 2 \int_{\pi}^{0} \sqrt{r^2 - r^2 cos^2(\phi)} (-rsin(\phi)) d\phi = -2r^2 \int_{\pi}^{0} sin^2(\phi)) d\phi - 2r^2 \int_{\pi}^{0} \left(\frac{1}{2} - \frac{1}{2} cos(2\phi) \right) d\phi = \pi r^2 \tag{1.30}$$

Comment: Trig sub gives expected answer.

Example (Bonus Fun - Coordinate System Conversion)

$$y_0 = \int_0^{\infty} e^{-x^2} dx = \int_0^{\infty} e^{-y^2} dy \tag{1.31}$$

Thus

$$y_0^2 = \int_0^{\infty} e^{-x^2} dx \int_0^{\infty} e^{-y^2} dy = \int_0^{\infty} \int_0^{\infty} e^{-x^2} e^{-y^2} dx dy \tag{1.32}$$

Using a rectangular to polar conversion yields

$$y_0^2 = \int_0^{\infty} \int_0^{\pi/2} e^{-r^2} r d\phi dr = \frac{\pi}{2} \left(-\frac{1}{2} \right) \int_0^{\infty} e^{-r^2} (-2r) dr = \frac{\pi}{4} \tag{1.33}$$

or

$$y_0 = \frac{\sqrt{\pi}}{2} \tag{1.34}$$

Comment: You can see why erf(x) has the $\frac{2}{\sqrt{\pi}}$ normalized factor in front - it's so that the area under the erf(x) is 1. This solution is a fun little trick that only works when the integral is extended to infinity. In general, though, if you see an integral whose limits are $-\infty$ to ∞, look to see if the integrand is an odd function (Hope Method). If it is, the integral is zero. Finally, notice that the area under the curve is finite, but that the length of the curve is infinite. This deserves some thought. If you had such a box, it would mean that you could fill the box with paint, but you would never have enough paint to paint the box!! Interestingly, It is also possible to have figures which have infinite perimeter with a finite area. Fractals have these properties.

1.4 Types and Methods of Solution

Since not all integrals have solutions in terms of Elementary functions, there are different types of solutions that may be obtained. Not all of these have equal merit.

1. Exact Analytical Explicit Solutions. (Best)
2. Exact Analytical Implicit Solutions. (Good)
3. Series Solutions. (Not as good)
4. Numerical Solutions. (OK, if nothing else works)

Students should familiarize themselves with the following solution methods.

1. Analytical Methods (such as the "Techniques of Integration", above).
2. Analytical and Numerical Solutions using Computer Algebra Systems (such as Maple).
3. Numerical Solutions using Spreadsheets.
4. Numerical Solutions using a programming language (such a C++).

As added motivation, students should be aware that

- These types and methods of solution apply to differential equations as well as integrals.
- Integrals are a special case of a differential equation.
- The laws of physics which apply to all of nature and devices created by man are governed by differential equations, **which, under many circumstances reduce to integrals**.
- Many of the algorithms for numerical integration and solution to differential equations are available (without charge) on the Internet (as is a host of related material). Try a web search for "Numerical Recipes in C++."

1.5 Applications of Calculus

There are very many applications of calculus. Below is a short table of functions and paramters of general interest to the mathematical and scientific community.

Application	Formula
Area under curve	$\int_a^b f(x)dx$
Slope of curve	$\frac{df}{dx}$
Extremum of a curve (max, min, inflection pt.)	$\frac{df}{dx} = 0$
Inflection point of a curve	$\frac{d^2f}{dx^2} = 0$
Arc Length	$\int_a^b \sqrt{1 + \left(\frac{df}{dx}\right)^2}dx$
Curvature (Radius)	$\frac{\left[1+\left(\frac{df}{dx}\right)^2\right]^{3/2}}{\left\vert\frac{d^2f}{dx^2}\right\vert}$
Average of a function	$\frac{1}{b-a}\int_a^b f(x)dx$
RMS of a function	$\sqrt{\frac{1}{b-a}\int_a^b f^2(x)dx}$
Center of Mass	$\int_a^b xf(x)dx$
Variance (Second Central Moment)	$\int_a^b (x-\bar{x})^2 f(x)dx$
Skewness (Third Central Moment)	$\int_a^b (x-\bar{x})^3 f(x)dx$
Kurtosis (Fourth Central Moment)	$\int_a^b (x-\bar{x})^4 f(x)dx$
Function Squared Norm	$\int_a^b \vert f(x)\vert^2 dx$
Fourier Transform	$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt$
Convolution Integral	$\int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau$
Taylor Coefficients	$\frac{1}{n!}\frac{d^n f}{dx^n}\vert_{x=a}$
Fourier Sine Series	$\frac{1}{\pi}\int_{-\pi}^{\pi} f(t)sin(nt)dt$
Fourier Cosine Series	$\frac{1}{\pi}\int_{-\pi}^{\pi} f(t)cos(nt)dt$