Name ______________________ Section ______________ Date ______________

Lab Partner(s) ____

EXPERIMENT 11

Archimedes' Principle: Buoyancy and Density

TI Advance Study Assignment

Read the experiment and answer the following questions.

1. Describe the physical reason for the buoyant force in terms of pressure.

2. Show that the buoyant force is given by $F_b = \rho_f g V_f$ using the development in the Theory section.

3. Give the conditions on densities that determine whether an object will sink or float in a fluid.

4. Distinguish between density and specific gravity, and explain why is it convenient to express these quantities in cgs units?

(continued)

5. Describe how the density of an object less dense than water can be determined using Archimedes' principle. How about the density of a liquid?

6. Why is it important to make certain that no air bubbles adhere to objects during the submerged weighing procedures? How would the experimental results be affected if bubbles were present?

Archimedes' Principle: Buoyancy and Density

Introduction and Objectives

Some objects float and others sink in a given fluid—a liquid or a gas. The fact that an object floats means it is "buoyed up" by a force greater than or equal to its weight. Archimedes (287–212 B.C.), a Greek scientist, deduced that the upward buoyant force acting on a floating object is equal to the weight of the fluid it displaces. Thus, an object sinks if its weight exceeds that of the fluid it displaces.

In this experiment, Archimedes' principle will be studied in an application: determining the densities and specific gravities of solid and liquid samples.

After performing this experiment and analyzing the data, you should be able to:

1. Tell whether an object will sink or float in a fluid, knowing the density of each.
2. Distinguish between density and specific gravity.
3. Describe how the densities of objects that sink or float may be determined experimentally.

Equipment Needed

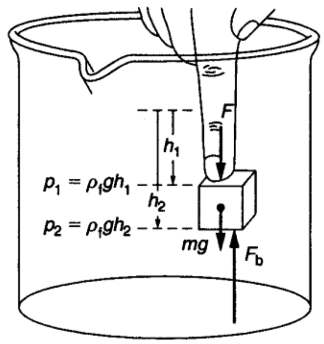
- Triple-beam pan balance with swing platform (or single-beam double-pan balance with swing platform and set of weights); see Fig. 2.
- Overflow can (or graduated cylinder and eye dropper)
- Two beakers
- Metal cylinder, irregularly shaped metal object, or metal sinker
- Waxed block of wood
- Saltwater solution or alcohol
- String
- Hydrometer and cylinder

Theory

When placed in a fluid, an object either floats or sinks. This is most commonly observed in liquids, particularly water, in which "light" objects float and "heavy" objects sink. But the same effect occurs for gases. A falling object is sinking in the atmosphere, whereas other objects float (● Fig. 1).

Things float because they are buoyant, or are buoyed up. That is, there must be an upward force that is greater

Figure 1 **Gas buoyancy.** Archimedes' principle applies to fluids—a liquid *or* a gas. Here, a helium-filled blimp floats in air. (Courtesy of Bill Aron/PhotoEdit.)



$\Delta p = \rho_f g(h_2 - h_1)$

Figure 2 Buoyancy. A buoyant force arises from the difference in pressure at different depths. The pressure at the bottom of the submerged block (p_2) is greater than that at the top (p_1), so there is a (buoyant) force directed upward (the arrow is shifted for clarity).

than (or equal to) the downward force of the object's weight. The upward force resulting from an object being wholly or partially immersed in a fluid is called the **buoyant force.** How the buoyant force arises can be understood by considering a buoyant object being held under the surface of a liquid (● Fig. 2). The pressures on the upper and lower surfaces of the block are given by the pressure-depth equations $p_1 = \rho_f g h_1$ and $p_2 = \rho_f g h_2$, respectively, where ρ_f is the density of the fluid. Thus there is a pressure difference $\Delta p = p_2 - p_1 = \rho_f g(h_2 - h_1)$, which gives an upward force (the buoyant force). In this case, the buoyant force is balanced by the applied force and the weight of the block.

It is not difficult to derive an expression for the magnitude of the buoyant force. If both the top and bottom areas of the block are A, the buoyant force (F_b) is given by $F_b = \Delta p A = \rho_f g V_f$, where V_f is the volume of the fluid displaced. But $\rho_f V_f$ is simply the mass of the fluid displaced by the block, m_f (recall that $\rho = m/V$). Hence the magnitude of the buoyant force is equal to the weight of the fluid displaced by the block. This general result is known as **Archimedes' principle:**

> *An object immersed wholly or partially in a fluid experiences a buoyant force equal in magnitude to the weight of the volume of fluid that it displaces.*

Thus the magnitude of the buoyant force depends only on the weight of the fluid displaced by the object, *not* on the weight of the object.

Archimedes' principle shows that an object

1. **will float** in a fluid if the density of the object ρ_o is less than the density of the fluid ρ_f that is, ($\rho_o < \rho_f$);
2. **will sink** if the object's density is greater than that of the fluid, ($\rho_o > \rho_f$) and
3. **will float in equilibrium** at any submerged depth where it is placed if its density is equal to that of the fluid, ($\rho_o = \rho_f$).

This can be shown mathematically as follows. The weight of an object is $w_o = m_o g = \rho_o g V_o$, where V_o is the volume of the object and $\rho_o = m_o/V_o$. Similarly, the weight of the fluid displaced by the object, or the buoyant force, is $F_b = w_f = m_f g = \rho_f g V_f$. If the object is completely submersed in the fluid, then $V_o = V_f$, and dividing one equation by the other yields

$$\frac{F_b}{w_o} = \frac{\rho_f}{\rho_o} \quad \text{or} \quad F_b = \left(\frac{\rho_f}{\rho_o}\right) w_o \tag{1}$$

Hence

1. If $\rho_o < \rho_f$, then $F_b > w_o$, and the object will be buoyed up to the surface and float.
2. If $\rho_o > \rho_f$, then $F_b < w_o$, and the object will sink.
3. If $\rho_o = \rho_f$, then $F_b = w_o$, and the object is in equilibrium.

Specific Gravity and Density

Specific gravity will be used in the study and determination of density. The **specific gravity** of a solid or liquid is defined as the ratio of the weight of a given volume of the substance to an equal volume of water:

$$\text{specific gravity (sp. gr.)} = \frac{w_s}{w_w}$$

$$= \frac{\text{weight of a substance (of given volume)}}{\text{weight of an equal volume of water}} \tag{2}$$

where the subscripts s and w refer to the substance and water, respectively.

Specific gravity is a density-type designation that uses water as a comparison standard. Since it is a weight ratio, specific gravity has no units. Conveniently, the numerical value of a substance's specific gravity is the same as the magnitude of its density *in cgs units.* This can be seen as follows:

$$\text{sp. gr.} = \frac{w_s}{w_w} = \frac{w_s/V_s}{w_w/V_w} = \frac{m_s g/V_s}{m_w g/V_w} = \frac{m_s/V_s}{m_w/V_w} = \frac{\rho_s}{\rho_w}$$

$$\text{sp. gr.} = \frac{\rho_s}{\rho_w} \tag{3}$$

Since, for practical purposes, the density of water is 1 g/cm³ over the temperature range in which water is liquid,

$$\text{sp. gr.} = \frac{\rho_s}{\rho_w} = \frac{\rho_s(\text{g/cm}^3)}{1\ (\text{g/cm}^3)} = \rho_s \quad \textbf{(4)}$$

where ρ_s is the numerical value of the density of a substance in g/cm³.

For example, the density of mercury is 13.6 g/cm³, and mercury has a specific gravity of 13.6. A specific gravity of 13.6 indicates that mercury is 13.6 times more dense than water, $\rho_s = (\text{sp. gr.})\rho_w$, or that a sample of mercury will weigh 13.6 times as much as an equal volume of water.

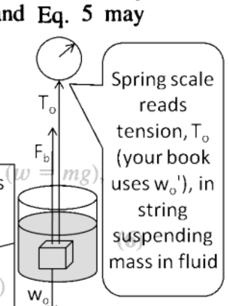
Archimedes' principle can be used to determine the specific gravity and density of a *submerged* object:

$$\text{sp. gr.} = \frac{w_o}{w_w} = \frac{w_o}{F_b} \quad \textbf{(5)}$$

where w_o is the weight of the object, w_w is the weight of the water it displaces, and, by Archimedes' principle, $w_w = F_b$.

For a heavy object that sinks, the net force as it does so is equal to $w_o - F_b$. (Why?) If attached to a scale while submerged, it would have a measured *apparent* weight T_O and $T_O = w_o - F_b$. Thus $F_b = w_o - T_O$, and Eq. 5 may be written

$$\text{sp. gr.} = \frac{w_o}{w_w} = \frac{w_o}{w_o - T_O}$$



~~or, in terms of mass measured on a balance ($w = mg$),~~

Mass of an object doesn't change, therefore, as we believe students may get confused, we will do this experiment strictly in terms of forces.

~~$$\text{sp. gr.} = \frac{m_o}{m_o - m'_o} = \rho_o \quad (6)$$~~

~~(of a heavy object that sinks)~~

~~where ρ_o is the magnitude of the density of the object in g/cm³.~~ This provides us with an experimental method to determine the specific gravity and density of an object that sinks.

To measure the specific gravity and density of an object that floats, or is less dense than water, using Archimedes' principle, it is necessary to use another object of sufficient weight and density to submerge the light object completely.

A scale measuring the sinker only lowered into the fluid would read $T_s = w_s - F_{bs}$

$T_1 = w_o + T_s$ is the measured weight ~~mass~~ of the object and the sinker, with only the sinker submerged, and $T_2 = T_O + T_s$ is the measured weight when both are submerged. Then

$$T_1 - T_2 = (w_o + T_s) - (T_O + T_s) = w_o - T_O = F_b$$

and the specific gravity and density can be found from Eq. 6. That is,

$$\text{sp. gr.} = \frac{w_o}{T_1 - T_2} \quad \textbf{(7)}$$

(of a light object that floats)

The specific gravity and density of a liquid can also be found using Archimedes' principle. A heavy object is weighed first in air (w_o) and then when submerged in liquid ($T_{O\ell}$). Then $(w_o - T_{O\ell})$ is the weight of the volume of liquid the object displaces, by Archimedes' principle. When we carry out a similar procedure for the object in water, $(w_o - T_{Ow})$ is the weight of the volume of water the object displaces.

Then, by the definition of specific gravity (Eq. 2),

$$\text{sp. gr.} = \frac{(w_o - T_{O\ell})}{(w_o - T_{Ow})} = \rho_\ell \quad \textbf{(8)}$$

(of a liquid)

where ρ_ℓ is the density of the liquid in g/cm³.

You may have been thinking that there are easier ways to determine the density or specific gravity of a solid or liquid. This is true, but the purpose of the experiment is to familiarize you with Archimedes' principle. You may wish to check your experimental results by determining the densities and specific gravities of the solid samples by some other method. The specific gravity of the liquid sample will also be determined using a hydrometer.

Experimental Procedure

A. *Direct Proof of Archimedes' Principle*

1. Weigh the metal sample and record its mass m_o and the type of metal in the Laboratory Report. Also, determine the mass of an empty beaker m_b and record. Fill the overflow can with water, and place it on the balance platform. Attach a string to the sample and suspend it from the balance arm (use a spring scale or calculate from a mass balance) as illustrated in ● Fig. 3.*
2. The overflow from the can when the sample is immersed is caught in the beaker. Measure the force, T_O of the submerged object. Make certain that no bubbles adhere to the object. (It is instructive to place the overflow can on a second balance, if available, and note that the "weight" of the overflow can does not change as the sample is submerged.)

* You may use an alternative method if no overflow can is available. Attach a string to the sample and place it in a graduated cylinder. Fill the cylinder with water until the sample is completely submerged. Add water (with an eyedropper) until the water level is at a specific reference mark on the cylinder (e.g., 35 mL). Remove the sample, shaking any drops of water back into the cylinder, and weigh the cylinder and water (m_b). Refill the cylinder to the reference mark and weigh it again ($m_w + m_b$). The mass of the "overflow" water is then the difference between these measurements.

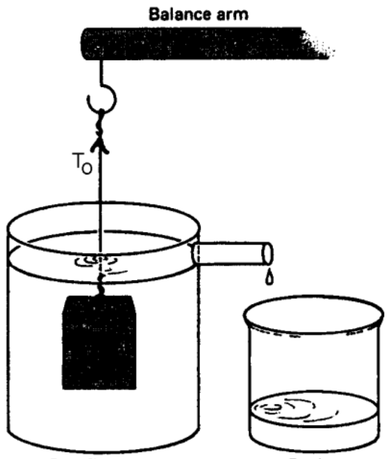


Figure 3 Archimedes' principle. The arrangement for proving Archimedes' principle. The weight of the displaced liquid that overflows into the beaker is equal to the reduction in weight of the metal sample when it is submerged, which is equal to the buoyant force.

Next weigh the beaker and water so as to determine the mass of the displaced water m_w (If the can does not fit on the balance platform, first suspend and immerse the object in the full overflow can, and catch the overflow in the beaker and find m_w. Then attach the sample to the balance arm and suspend it in a beaker of water that will fit on the balance platform to find T_O.)

3. The buoyant force is then the difference between the object's true weight and the weight reading when submerged, $F_b = m_o g - T_O$. According to Archimedes' principle, the magnitude of the buoyant force F_b should equal the weight of the displaced water:

$$F_b = w_w = m_w g$$

or

$$F_b = m_o g - T_O = m_w g$$

Compute the buoyant force, and compare it with the weight of the displaced water by finding the percent difference.

B. Density of a Heavy Solid Object ($\rho_o > \rho_w$)

4. Determine the specific gravity and density of the metal sample. This can be computed using the data from part A.

C. Density of a Light Solid Object ($\rho_o < \rho_w$)

5. Determine the specific gravity and density of the wooden block by the procedure described in the Theory section. First, measure the mass of the wooden block alone (in air). Then set up as in Fig. 3.

   Tie the sinker to the wood block, and tie the block to the lower hook of the balance. With the beaker empty, check that the sinker does not touch the bottom of the beaker and that the top of the wooden block is below the top of the beaker. Pour enough water into the beaker to cover the sinker, weigh, add more water until the wooden block is submerged, and then weigh again. Make certain that no air bubbles adhere to the objects during the submerged weighing procedures. The block is waxed so that it does not become waterlogged.

D. Density of a Liquid (ρ_ℓ)

6. Determine the specific gravity and density of the liquid provided, by the procedure described in the Theory section. Again, make certain that no air bubbles adhere to the object during the submerged weighing procedures.

7. Determine the specific gravity of the liquid using the hydrometer and cylinder. Compare this value with that found in procedure 6 by computing the percent difference.

Name ______________________ Section ______________ Date ______________

Lab Partner(s) ____

EXPERIMENT 11

Archimedes' Principle: Buoyancy and Density

TI Laboratory Report

A. Direct Proof of Archimedes' Principle

Type of metal ______________

Mass of metal m_o in air ______________

Mass of beaker m_b ______________

Force reading of metal submerged in water, T_o ______________

Mass of beaker and displaced water $m_w + m_b$ ______________

Mass of displaced water m_w ______________

Buoyant force (in newtons) ______________

Weight of displaced water (in newtons) ______________

Percent difference ______________

Calculations
(show work)

Don't forget units

(continued)

B. Density of a Heavy Solid ($\rho_o > \rho_w$)

Calculations
(show work)

Specific gravity ________

Density ________

C. Density of a Light Solid ($\rho_o < \rho_w$)

Mass of block in air ________

Force reading of block and sinker when only the sinker is submerged, T_1 ________

Force reading of block and sinker when both are submerged, T_2 ________

Specific gravity ________

Density ________

Calculations
(show work)

Name ______________________ Section ______________ Date ______________

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D. Density of a Liquid (ρ_ℓ)

Mass of object in air ______________

Force reading of object submerged in unknown liquid, T_ℓ ______________

Force reading of object submerged in water, T_W ______________

Computed sp. gr. ______________

Sp. gr. from hydrometer measurement ______________

Percent difference ______________

Calculations
(show work)

TI QUESTIONS

1. Look up the density of the metal of the object used in parts A and B of the procedure, and compare it with the experimental value. Comment on the purity of the metal of the object. (Archimedes developed his principle while working on a similar inquiry. His problem was to determine whether a crown alleged to be made of pure gold had actually been made with some content of cheaper metal.)

(continued)

2. In part B, the string will cause error. When does it lead to an experimental density that is too high? Too low?

3. Discuss the situation that occurs when an object is immersed in a fluid that has the same density as the object.

4. (a) Explain how a submarine is caused to submerge and surface without the use of its propulsion propeller and fins.

   (b) Which is heavier, a given volume of ice or the same volume of water? Justify your answer.

5. A block of wood floats in a beaker of water. According to Archimedes' principle, the block experiences an upward buoyant force. If the beaker with the water and floating block were weighed, would the measured weight be less than the sum of the weights of the individual components? Explain.

6. A person can lift 45 kg (≈ 100 lb). Using the experimental value of the specific gravity for the metal object in part B, how many cubic meters of the metal could the person lift (a) in air, (b) in water? How many actual kilograms of metal is this in air, and in water?

7. Explain the principle and construction of a hydrometer. What is the purpose of the common measurements of the specific gravities of an automobile's radiator coolant and battery electrolyte?