

Lab 1 – Fundamentals

Purpose

In this lab we will study background skills needed for future labs including scientific notation, units and dimensions, dimensional analysis, SOLVE, significant digits or significant figures (sig. figs.), random and systematic errors, accuracy and precision, mean and standard deviation, using measurement tools, graphing, transforming data, and the best fit straight line.

Equipment

- Pencil
- Paper
- Calculator
- Ruler

Introduction

The scientific method is based on quantitative measurement and observation and determination of whether or not those measurements and observations fit our theoretical understanding. The first thing to understand is how accurate and precise is our measurements and observations. If the theory is within experimental error we conclude that the experiment confirms theory. If not we must first inspect the experiment to assess if it is actually measuring what we desire. We must also inspect our calculations to insure their correctness. If we have done a good experiment and our calculations are accurate then, and only then, might we consider a revision to the theory.

The vast majority of the time there is an experimental problem rather than a problem with the underlying theory, however the exceptions are monumental. One such exceptional experiment was the Michelson and Morley experiment where they attempted to measure how fast the Earth traveled through the medium that carries light wave. They knew Earth traveled at different velocities in its orbit. So at different times the Earth would move through this medium with different speeds. It would be like an airplane with a headwind instead of a tail wind. What they found was no change in velocity. Michelson and Morley thought their experiment was a failure. Only later was it learned that they did a very good experiment which laid the foundation for Einstein's Theory of Relativity.

Throughout your studies we are going to work on improving your experimental skills so you too can perform very good experiments. We need to design our experiment and choose measurement instruments. Then there are a plethora of techniques to learn to prevent mistakes (SOLVE, scientific notation, proper use of dimensions, dimensional analysis), minimize errors (sig. figs., mean, standard deviation), and analyze data (graphing, data transformation, best fit line).

This is only the beginning. In your future courses you will learn much more about these things, however this introductory lab should provide you will some grounding to assist you in your future studies.

Theory

A. Scientific notation

The Hindu-Arabic number system, the system the whole world now uses, is a place value system. It was first invented in India and came to Europe through the Middle East – hence the name Hindu-Arabic. The Babylonians also invented a place value system based on the number 60. We still see remnants of that today in 60 seconds in a minute and 60 minutes in an hour. The problem was there are 3600 addition facts and 3600 multiplication facts in their base 60 system. The Hindu-Arabic system is base 10 – only 100 addition facts and 100 multiplication facts. The Mayans invented a base 20 system independently and unknown to the “old world,” but the Hindu-Arabic system was well established in Europe when the Mayan civilization was discovered. If Native Americans invaded Europe instead of the other way around, we might be using a base 20 number system today. So the base 10 system caught on.

The “old world” used systems similar to Roman Numerals prior to the Hindu-Arabic system. Counting 1, 2, 3, etc. in Roman Numerals would go like this: I, II, III, IV, V, VI, VII, VIII, IX, X, XI, etc. It was confusing. If the letter standing for one (I), came before the letter standing for 5 (V) you subtract, otherwise you add. It got even more confusing and complicated for larger numbers. How do you write 1999 in Roman Numerals? Is it MIM, MCMIC, MCMLIL, MCMLXLIX, or MCMLXLVIV? M was also the largest number (1000) and it couldn't handle fractions. It's easy to understand why our modern decimal system caught on so quickly and why Roman Numerals were rapidly replaced.

In our decimal system there are only 10 digits – 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. Unlike Roman Numerals where each symbol has a specific value, in our decimal system the digit and the *place* in the number both determine the value. That's why we call it a place value system. Also there's a thing called a decimal point. Digits to the left of the decimal point have values greater than one and digits to the right represent values less than one. So 1987.654 is one thousand, 9 hundred, 8 tens, 7 ones, 6 tenths, 5 hundredths, and 4 thousandths.

Except for the historical context, this information should not be new to you. You should be very familiar with it. However scientists began to encounter difficulties and had to invent scientific notation. And when calculators were introduced in the early 1970s we had even more modern problems.

Why do we need scientific notation? An example – the mass of the Earth without scientific notation is 5,980,000,000,000,000,000,000 kg. With scientific notation it is 5.98×10^{24} kg. Just like we had to get used to place values, the advantages of scientific notation far outweigh the difficulty.

How does scientific notation work? Basically we turn a number into a product of two numbers. The first is a number between 1 and 10 and the second is a power of 10. Let's examine the mass of the Earth again.

$$5,980,000,000,000,000,000,000 \text{ kg} = (5.98) \times (1,000,000,000,000,000,000,000) \text{ kg.} \quad \text{Equation 1.1}$$

We can now use exponential notation for 1,000,000,000,000,000,000,000. In exponential notation $1,000,000,000,000,000,000,000 = 10^{24}$ (exponents should also be very familiar to you). Substituting this into Equation 1.1 and you get:

$$5,980,000,000,000,000,000,000 \text{ kg} = 5.98 \times 10^{24} \text{ kg.} \quad \text{Equation 1.2}$$

We can write small numbers using scientific notation also using negative exponents. The charge on the electron is 1.602×10^{-19} Coul where $10^{-19} = \frac{1}{10,000,000,000,000,000,000}$.

Note – wherever possible use a straight fraction bar, for example, $\frac{1}{10,000,000,000,000,000}$ instead of a slanted fraction bar, for example, $1/10,000,000,000,000,000$. The straight fraction bar is easier to read, understand, and perform calculations without mistakes. And we want to perform calculations without mistakes! Occasionally we'll use the slanted fraction bar, but only when the formula is simple and unequivocal. For anything even remotely complex we'll use the straight fraction bar to avoid misunderstanding. You should do this too! As we'll see next, however, calculators make this difficult so we have to be smart about using those marvelous calculators. And if you don't think calculators are marvelous, I'll let you try my slide rule. Like any tool, though, we need to understand them to use them without mistakes.

When calculators came along in the 1970s we encountered another problem – how do we tell calculators how to do scientific notation? Let's divide 1×10^{-4} by 1×10^{-4} . A non-zero number divided by itself is simply one – right? Let's try it. Do this on your calculator. The lab procedure asks you to do this – do it now or do it later.

Use these key strokes on your scientific calculator (your calculator may have minor variations to this): $\boxed{1} \boxed{\times} \boxed{1} \boxed{0} \boxed{y^x} \boxed{-} \boxed{4} \boxed{\div} \boxed{1} \boxed{\times} \boxed{1} \boxed{0} \boxed{y^x} \boxed{-} \boxed{4} \boxed{=}$. Does this equal one? Probably not. The way most calculators do order of operations the answer will come out 1×10^{-8} or 0.00000001 instead of 1. We have got to tell the calculator to do the order of operations correctly for scientific notation.

One more thing. On many calculators and most computers the $\boxed{\div}$ button is labeled $\boxed{/}$, in other words, our enemy the slanted fraction bar. Calculators and computers love it because that's their language, but homo sapien's language is the straight fraction bar. Use the straight fraction bar and only convert when you're "plugging and chugging," that is, entering numbers and formula into calculators and doing calculations. And "plugging and chugging" should be the very last step in solving a problem.

A student of mine once quipped, "We need a special button for this." Engineers in the 1970s invented that button! On different calculators it's labeled different things so you'll need to refer to your calculator's instructions. Here's a few of the labels: \boxed{E} , \boxed{EE} , $\boxed{\times 10^x}$, and, especially the scientific calculator built into the Windows operating system, \boxed{Exp} . From now on I'll use \boxed{EE} to stand for this special button because that's the most common.

Let's try our division problem again, but this time use our special button, and see if the answer is now 1. Enter $\boxed{1} \boxed{EE} \boxed{-} \boxed{4} \boxed{\div} \boxed{1} \boxed{EE} \boxed{-} \boxed{4} \boxed{=}$. The answer should equal 1 as we expect.

Try a different problem now: $[1 \times 10^{-4}] \times [1 \times 10^4]$. Enter the numbers in the square brackets, [and], using your calculator's scientific notation. The answer to this should also equal 1, but if your answer is 100 then there's another problem using the calculator. If you got an answer of 100 did you enter $\boxed{1} \boxed{0} \boxed{EE} \boxed{-} \boxed{4} \boxed{\times} \boxed{1} \boxed{0} \boxed{EE} \boxed{4} \boxed{=}$? Some calculators call the \boxed{EE} button $\boxed{\times 10^x}$. $\boxed{\times 10^x}$ is a better reminder of what the \boxed{EE} button stands for. If you enter $\boxed{1} \boxed{0} \boxed{EE} \boxed{-} \boxed{4}$ you're actually entering the "10" twice. The number you entered is 10×10^{-4} . And if you multiply it by $10 \times 10^{+4}$ of course the answer will be 100 and not 1. To do this multiplication problem correctly enter $\boxed{1} \boxed{EE} \boxed{-} \boxed{4} \boxed{\times} \boxed{1} \boxed{EE} \boxed{4} \boxed{=}$.

B. Units and dimensions

You know how money has the faces of dead presidents? Well, units in the SI system (Standard International – the metric system in use today) are named after dead scientists – sometimes.

Only one of the fundamental units is named after a dead scientist – the Coulomb (it's the unit of electrical charge). Electricity was discovered after the metric system was invented and the description of electrical phenomena was very complicated in the older metric system. For example, the unit of charge was a length unit times the square root of a force. Fortunately the English system never tried to deal with electricity and for the SI system we figured out that electrical charge was as fundamental as the length, time, and mass. SI therefore avoids and eliminates those pesky square roots of force and other pesky, weird, and difficult to understand units. To give you an idea of how much a Coulomb is, an Ampere is a Coulomb per second of electrical current. And 15 Amperes is typically delivered by a standard wall outlet.

The other fundamental units are meter, kilogram, and second – none of these are the names of dead scientists. The meter, m, is a unit of length and one inch is exactly equal to 0.0254 m. By convention this is the agreed upon conversion between inches and meters.

The kilogram, kg, is a unit of mass. The conversion to the English systems of units, similar to the meter, is defined by convention: The mass of a one pound object equals 0.45359237 kg. There was a lot of historical confusion between mass and force, but we'll deal with that later. The kilo in kilogram is a prefix standing for 1000 which means a kg = 1000 grams. We'll have more on prefixes later. Note that to prevent confusion between acceleration of gravity, g, and a unit of a gram, the gram is often abbreviated gm.

The second is also a fundamental unit and, fortunately, the English second is the same as the metric second.

Temperature is sort of fundamental. It's connected with energy, but in a complicated way buried in the theory of thermodynamics. Practically speaking it's a fundamental unit.

There are three temperature scales in common use. The scale used in the USA is called the Fahrenheit scale named after Daniel Fahrenheit. On that scale water freezes at 32°F and boils at 212°F. This is a good scale for weather since at temperature latitudes normal temperature ranges from about 0°F and 100°F.

The Celsius scale (named after Anders Celsius) is good for science since 0°C is the freezing point of water and 100°C is the boiling point. Since most labs have water, you can make a thermometer that reads 0 when water freezes and 100 when it boils. The Celsius scale was formerly referred to as the centigrade scale. Centi means 100 (100 cents in a dollar, 100 years in a century, 100 centimeters in a meter, and percent means parts per 100) and Celsius divided the temperature between the freezing and boiling of water into 100 parts.

The third scale is the Kelvin (named after a knighted scientist, Lord Kelvin – William Thomson) temperature scale and it measures absolute temperature. You can have negative Celsius and Fahrenheit temperatures, but never a negative Kelvin or absolute temperature. 0°K is the theoretical temperature where classically an object's atoms stop moving (we'll deal with quantum effects later) and it equals -273.15°C. The Kelvin scale is also closely related to the Celsius scale since a change of 1°K equals a change of 1°C. The conversion between T_k and T_c (temperature Kelvin and temperature Celsius) is simply: $T_c = T_k + 273.15^\circ\text{C}$. Other conversions are a little more complicated and we'll discuss those later.

All other units are derived. For example, a Newton, N, is a unit of force and is a $\text{kg m} / \text{sec}^2$. A Joule is a unit of energy and equals a N m. And so on. Later we'll talk about dimensional analysis. For example, in a formula if the unit is a Joule on the left hand side of the equal sign, our units better work out to be Joules on the right hand side of the equal sign. If this isn't true we've probably made a mistake and need to look for errors.

C. Unit Conversion

Before we get into conversions, let's list what the prefixes mean. We've already encountered two – kilo and centi. We won't list all of them – just the most commonly used. For example, deci, deca, and hector have largely fallen into disuse. If you need these you can look them up when you need them. As we'll see shortly, centi is also a little unusual, but in such common usage that it's unlikely to fall into disuse anytime soon.

Prefix	Abbreviation	Meaning	Examples
femto	f	10^{-15}	fs = femtosecond, used in some very fast experiments
Pico	p	10^{-12}	pF = picoFarad, common capacitance value in circuits
nano	n	10^{-9}	ns = nanosecond, nm = nanometer, the wavelength of visible light ranges from about 400 nm to 700 nm
micro	μ (Greek letter mu)	10^{-6}	μm = micrometer or micron. A common tool, the micronometer, can measure dimensions to a μm
milli	m	10^{-3}	mm = millimeter, ms = millisecond
centi	c	10^{-2}	cm = centimeter
Kilo	k	1000	km = kilometer, kg = kilogram
Mega	M	10^6	MW = megawatt
Giga	G	10^9	GW = gigawatt, Gb = gigabyte, GHz = gigaHertz. Pronounced with a hard G – not J sound as “Doc Brown” did it in “Back to the Future.”
Tera	T	10^{12}	Tb = terabyte. These big numbers commonly refer to computing

Note that the commonly used prefixes go by factors of 1000. centi is the only exception, however cm is so common that it will probably be with us for a very long time.

Let's do a few examples. 10^{-15} sec = fs. This is a statement of a units conversion fact. Multiplying both sides by 10^{15} and we get an equivalent conversion fact: 1 sec = 10^{15} fs. How many fs are in 7 sec? When doing units conversion always use the unit fraction as follows:

$$(7 \text{ sec}) \left(\frac{10^{15} \text{ fs}}{\text{sec}} \right) = 7 \times 10^{15} \text{ fs} \quad \text{Equation 1.2}$$

The fraction, $\left(\frac{10^{15} \text{ fs}}{\text{sec}} \right)$, is called a unit fraction. Notice in Equation 1.2 that the unit of sec in the numerator cancels the unit of sec in the denominator. If this happens then we're confident we've done our conversion correctly. If not, then we have to go back and find our mistake. Get into the habit of using unit fractions.

We can use unit fractions sequentially. For example, let's convert 10 km into ft. We'll need another fact: 3.28 ft = 1 m.

$$(10 \text{ km}) \left(\frac{1000 \text{ m}}{\text{km}} \right) \left(\frac{3.28 \text{ ft}}{\text{m}} \right) = 32,800 \text{ ft}$$

D. Dimensional analysis

Dimensional analysis may include applying formulas PLUS units conversion. It's very important to use to insure our results are correct. ALWAYS use dimensional analysis. An example is applying the equations

$F = ma$ where F is force in Newtons ($N = \frac{\text{kg m}}{\text{sec}^2}$), m is mass in kg, and a is acceleration in $\frac{\text{m}}{\text{sec}^2}$. What is the force of gravity ($a = 9.8 \frac{\text{m}}{\text{sec}^2}$) in Newtons on a 500 gm mass?

$$F=ma=(500 \text{ gm}) \left(9.8 \frac{\text{m}}{\text{sec}^2}\right)=4900 \frac{\text{gm m}}{\text{sec}^2} \quad \text{Equation 1.3}$$

Is the answer ($4900 \frac{\text{gm m}}{\text{sec}^2}$) the correct or final answer? No, and that's easy to see by inspecting units. $\frac{\text{gm m}}{\text{sec}^2}$ is not a Newton – a Newton is a $\frac{\text{kg m}}{\text{sec}^2}$. The final step is to convert gm to kg.

$$\left(4900 \frac{\text{gm m}}{\text{sec}^2}\right) \left(\frac{\text{kg}}{1000 \text{ gm}}\right) = 4.9 \text{ N}$$

By the way, if we converted 500 gm to 0.5 kg first then Equation 1.3 would become:

$$F=ma=(0.5 \text{ kg}) \left(9.8 \frac{\text{m}}{\text{sec}^2}\right)=4.9 \frac{\text{kg m}}{\text{sec}^2}=4.9 \text{ N} \quad \text{Equation 1.4}$$

If you anticipate needing to do units conversions, it's highly advisable to perform all conversions before calculations.

E. SOLVE

Forgive me Mr. Lucas for altering your iconic image of Obiwan Kenobi to tell students to **use the SOLVE method**.



Always, always use the SOLVE method. Refer to <http://iws.collin.edu/jtsizemore/help/fivestep.htm>. In summary SOLVE stands for:

- 1) **S - Sketch:** Sketch, draw a picture, understand the problem
- 2) **O - Organize:** Organize, write down known and unknown quantities
- 3) **L - List:** List relevant equations, determine which are applicable
- 4) **V - Vary:** Vary, rewrite, and transform equations to express unknown quantity in terms of known quantities
- 5) **E - Evaluate:** Evaluate expressions, Plug in Numbers, evaluate to determine if answer makes

One variation is found at <http://pages.cms.k12.nc.us/amyryan/solve.html> and comes from the textbook Foundations of Algebra. K-12 teachers are starting to use this more and more consistently as time goes on. Why, because it works! The differences between our version and the "Foundations of Algebra" version are minor. The headline is to get in the habit – pick whichever version you like and **use the SOLVE method**. Your instructor will consistently, by example, show you how to apply this SOLVE method through this term. Imitate your instructor and, as Obiwan says, "**Use the SOLVE Method Luke.**" Here's the Foundations of Algebra version:

The SOLVE Method:

S: Study the problem. This is referred to as "S" the problem. The first step is to highlight the question. The next step is to answer the question "What is the problem asking me to find?" Always start the answer to this question with the words "I need to find."

O: Organize the facts. This step is referred to as "O" the problem. You must always "S" the problem first, then you may "O" the problem. There are three steps to "O." 1) Identify the facts by circling each fact. 2) Eliminate unnecessary facts by crossing them out. 3) List the necessary facts.

L: Line up a plan. Students must "S" and "O" the problem before they "L" the problem. In this steps students need to choose the operation(s) or plan they will use to solve the problem. They must use the facts listed in "O" when writing their plan. No numbers please - just word phrases.

V: Verify your plan with action. Students will use the verbal expressions in the "L" step to write a numerical expression or equation that can be solved. This is where they actually do the math and find the answer.

E: Examine the results. In this step, students will ask the questions: "Did you answer what you were asked to find in S?" "Is my answer accurate?" "Is my answer reasonable?" To answer these questions, students must check their work by reworking the problem, working out the problem using another method, or working backwards. Next, students check the answer by estimating. The final step is to write the answer with its unit.

F. Significant digits or significant figures (sig. figs.)

"Sig figs" are a way to describe the accuracy of a number. In the old days we had slide rules and the calculation tool (the slide rule) was only accurate to two sig figs. The old joke was, two times two is 3.9 and that's close enough. Today we use scientific calculators which are commonly accurate to 13 sig figs – way more than needed. We don't have to worry about the limitations of our calculators anymore.

There are complicated rules for rounding numbers to a certain number of sig figs. We're going to let you research this and give you the short version. To round a number to n sig figs:

- 1) Enter the number into your calculator
- 2) Convert the number into scientific notation keeping at least $n+1$ digits
- 3) Round the number to n digits
- 4) Optionally convert the rounded number back to standard notation

For example, round the numbers in the following table to 3, 4, and 5 sig figs. Recall the "E" stands for "times 10 raised to the power of." So $3.77507\text{E}-13 = 3.77507 \times 10^{-13}$. Also most computer and calculators require you to enter scientific notation numbers using the "E" format. In other words, you would enter 3.77507×10^{-13} into a computer as 3.77507E-13. In the next row of this table we show the numbers converted to scientific notation. Since we're rounding to a maximum of 5 sig figs we need to keep 6 digits. Now round to 3, 4, or 5 digits.

Round just like you did in grade school. For example, to round 9.87531E-03 to 3 sig figs count 3 digits. "9" is the first digit, "8" is the second digit, and "7" is the third digit. If we're rounding to n sig figs, look at the $(n+1)$ th digit. If the $(n+1)$ th digit is 5 or greater, increase the n th digit by 1, otherwise do nothing. The last step

is to make all digits to the right of the n th digit zero. Using our example, the $(n+1)$ th (fourth digit since $n = 3$) of $9.87531\text{E}-03$ is “5.” So we add 1 to the n th (third) digit and turn digits to the right of the n th digit to zero. and $9.87531\text{E}-03 \approx 9.88\text{E}-03$. The wavy equal sign, \approx , means approximately equal to and it’s a good symbol to use when rounding.

	Numbers				
Original	0.009875306	9,909,932,731	3.77507E-13	7,161,233,190	3.15088E-15
Scientific	9.87531E-03	9.90993E+09	3.77507E-13	7.16123E+09	3.15088E-15
Sig figs rounded to:					
3	9.88E-03	9.91E+09	3.78E-13	7.16E+09	3.15E-15
4	9.875E-03	9.91E+09	3.775E-13	7.161E+09	3.151E-15
5	9.8753E-03	9.9099E+09	3.7751E-13	7.1612E+09	3.1509E-15

There are some subtleties. Rounding $9.90993\text{E}+09$ to 4 sig figs means adding 1 to 9, the fourth digit. Indeed, you add – you don’t just increase 9 to zero. You add and carry the 1 to the third digit. Also $9.90993\text{E}+09$ rounded to 3 sig figs is the same number as $9.90993\text{E}+09$ rounded to 4 sig figs, that is, $9.91\text{E}+09$. But when you look at this number you think it’s rounded to 3 sig figs. In case of confusion it is the responsibility of the writer to inform the reader of the accuracy of the number. The writer needs to clarify by writing, for example, “The answer is $9.91\text{E}+09$ and is accurate to 4 sig figs.”

G. Random and systematic errors, accuracy and precision

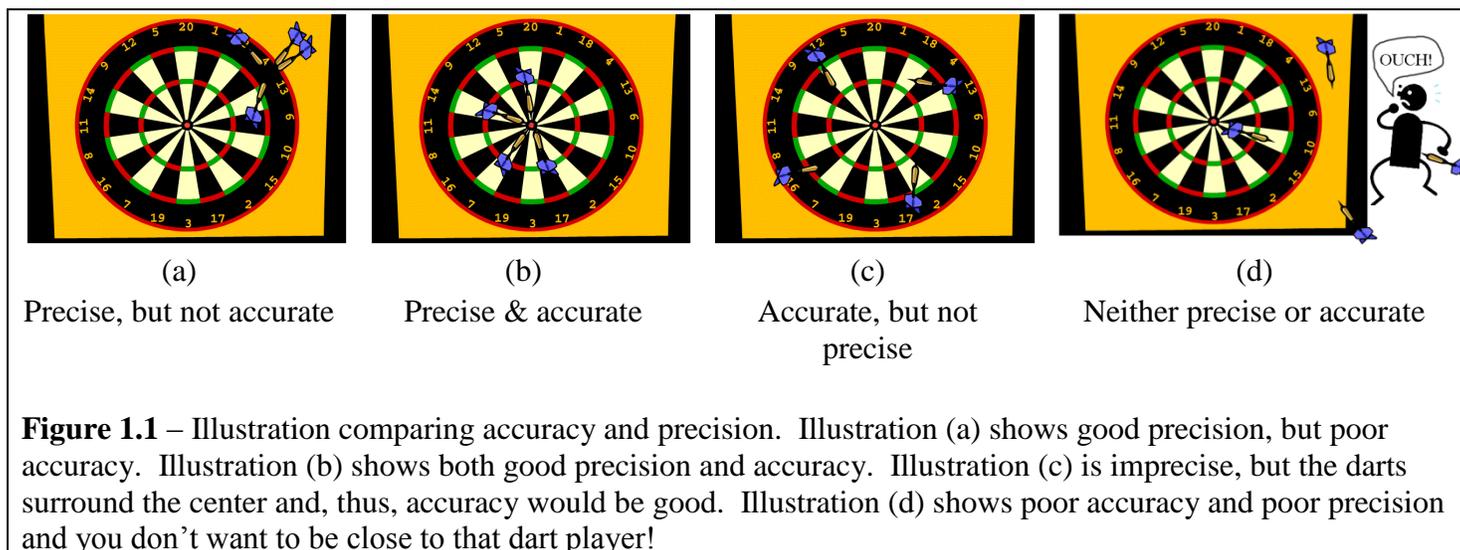
Random errors are things that statistically fluctuate. It may be due to variations between different people measuring the same thing. This is called imprecision. *Precision* has to do with our ability to repeat a measurement and how close the second measurement is to the first. Random errors can also have a natural origin. Identical twins have different fingerprints. So “Smallville” (a TV series about Superman, season 10, “Scion” episode) was wrong! Lionel Luthor from the alternate universe would have completely different fingerprints than Lionel Luthor from our universe – the fingerprints wouldn’t simply be mirror images of each other. Identical corn seed with identical DNA in the same soil, watered and fertilized the same way will grow to slightly different heights. Random errors are unpredictable and *indeterminate*, however they are likely to be both too large or too small about equally.

There are two basic ways to deal with random error. We can use the mathematics of statistics to accurately characterize it. More on this later. Or, as Ernest Rutherford (a farm boy from New Zealand who became arguably the greatest experiment physicist of all time) is quoted as saying, “If your experiment needs statistics, you ought to have done a better experiment.” In other words, use a more precise measuring tool. Instead of a hand held stopwatch, use a photogate. Or if you’re stuck with a stopwatch, time things falling from the top of a building instead or falling a couple meters. Experimentalists must be quite creative to minimize errors. We seek for you to acquire some of this skill.

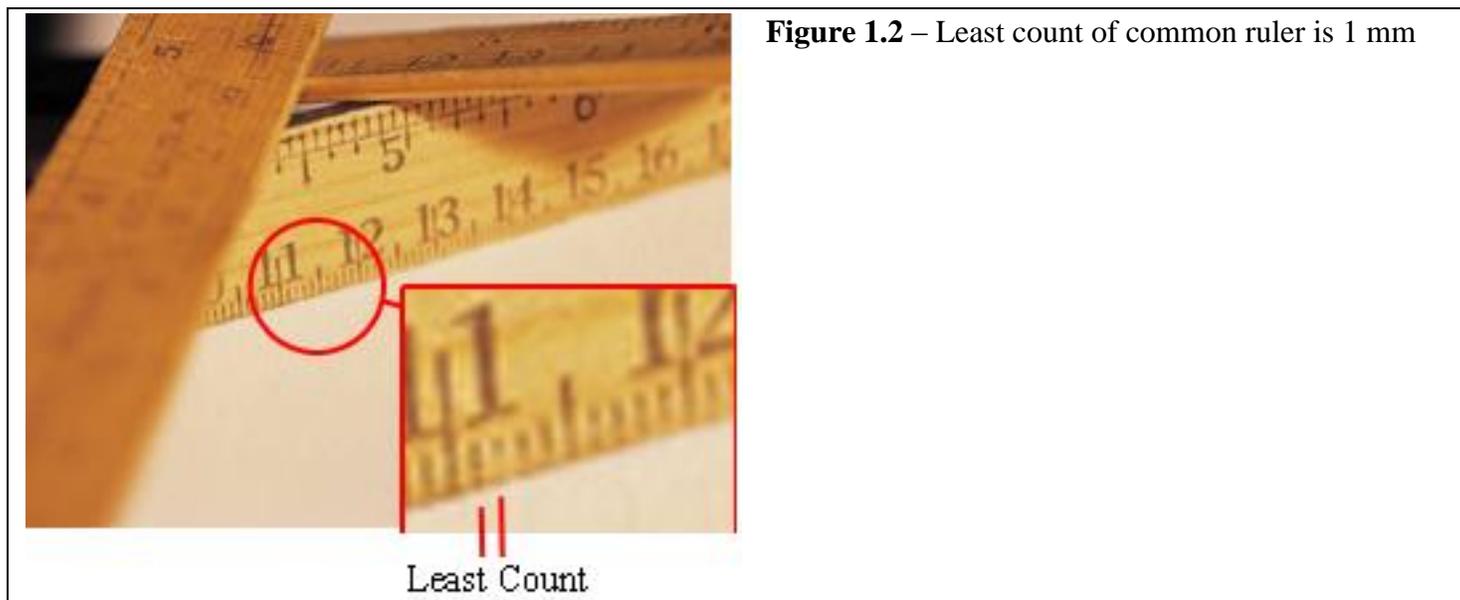
Systematic errors, on the other hand, are errors that have the same effect all the time. A mass balance not zeroed is a good example. In general any un-calibrated measuring tool can result in a systematic error. The headline here is to zero the mass balances and calibrate your measurement tools to the best of your ability. Sometimes it isn’t possible to zero a measurement tool. For example, when you fully close a micrometer it may read $3\ \mu\text{m}$. So you have to subtract $3\ \mu\text{m}$ from all measurements. This, by the way, is a *determinate* error. You can determine the source of the error and correct for it. Sometimes we may be able to identify a source of error, but not be able to correct for it. A string used to hold a mass adds a little mass, but you might not be able to determine how much. Calculation mistakes are also systematic errors and you need to do the best you can to eliminate those. Systematic errors cause *inaccuracies*. *Accuracy* measures how close you are to an accepted

value. This forms a mnemonic device: Accuracy refers to accepted value – both words start with “a” and have a short “a” sound (this is important later).

The following figure illustrates the distinction between accuracy and precision.



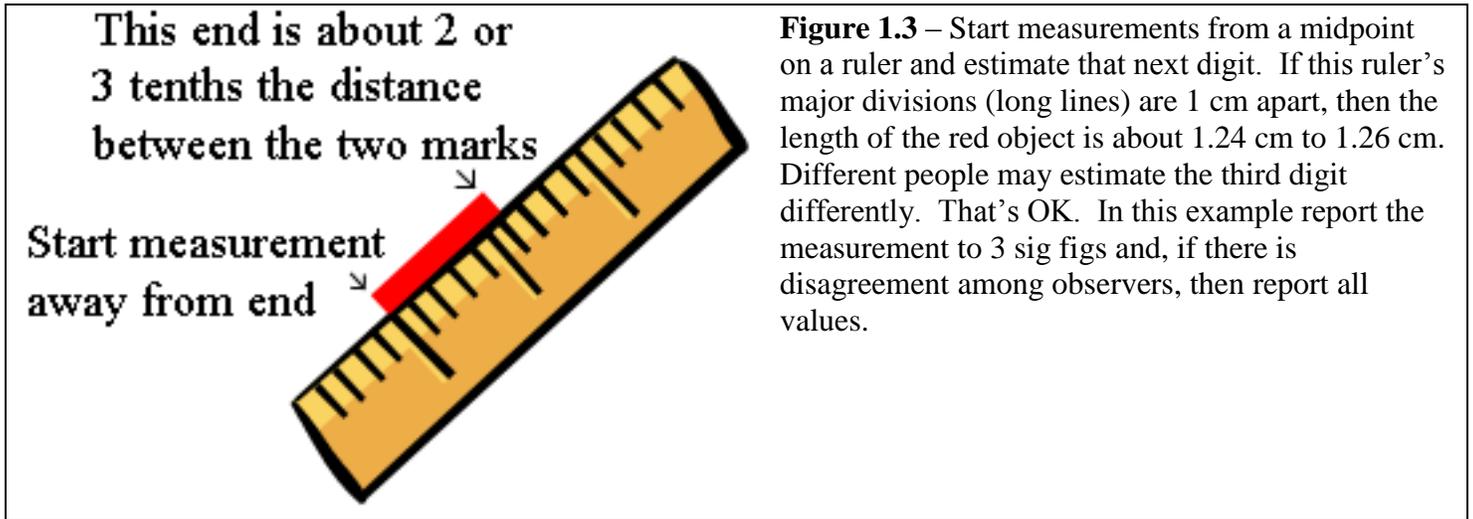
Closely related to accuracy and precision is *least count*. It’s simply the smallest division of a measuring instrument. For example, common rulers have a least count of 1 mm. Refer to Figure 1.2.



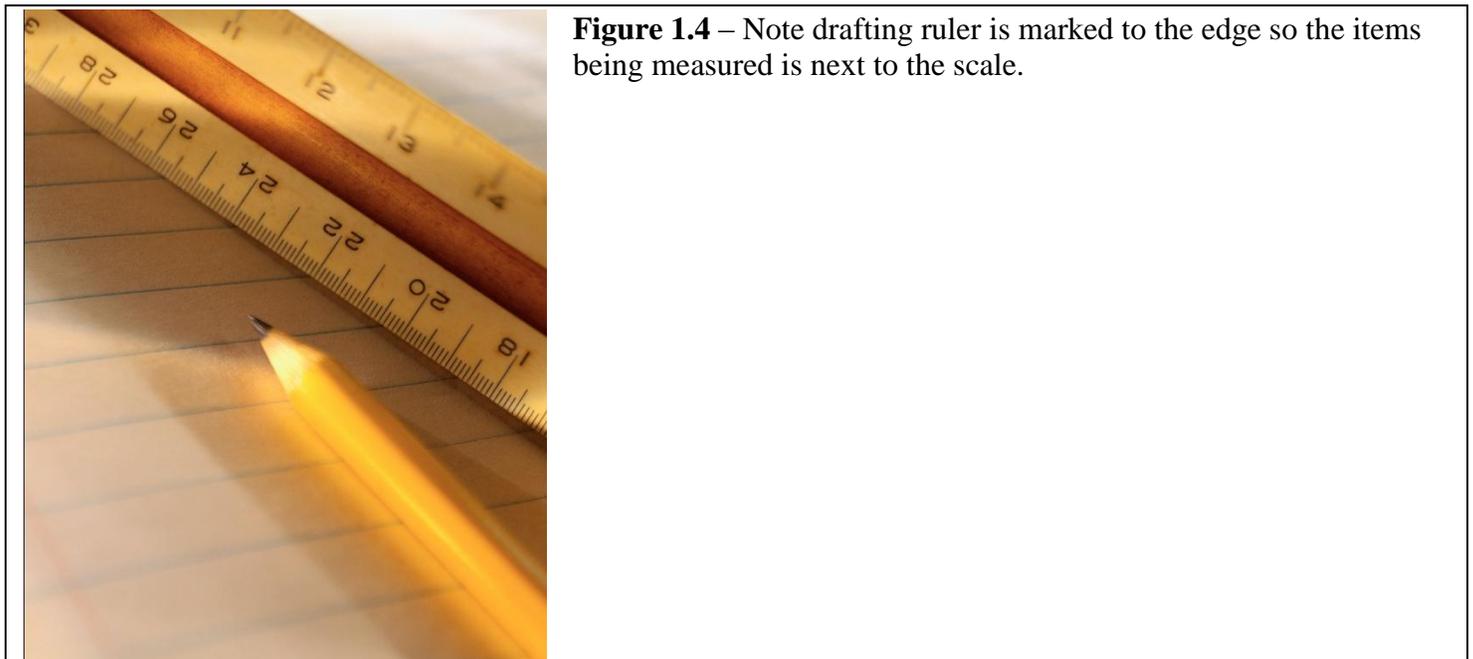
The precision of an answer to a calculation depends on the precision of the number that go into the calculations. If you multiply or divide, the answer is only precise to the sig figs of the least accurate number. For example, $9.8 \times 123 = 1205.4$, but this only precise to 2 sig figs (the sig figs of 9.8). Thus 1205.4 is only precise to 2 sig figs and, thus, you may report 1200 with a note “precise to 2 sig figs”. We need the note because 1200 without a note may mean 2 sig figs precision, 3 sig figs precision, or 4 sig figs precision. To avoid confusion, make a note. Also, keep 3 sig figs or more. We want you to report the answer as 1210 with a note, “accurate to 2 sig figs”. Remember, area and volume are derived units so it’s necessary to understand how the precision of a factor (or dividend or divisor or addend or subtrahend) influences the precision of the outcome.

When adding or subtracting the answer is precise to only the worst least count. $21 + 273.15 = 293.15$, but since the least count of 21 is only 1, it is only legitimate to keep the answer precise to the units digit. Therefore you only need report 293 as the answer.

A few more pointers. Start a measurement from a point along the scale and not at the end of a ruler. The end of a ruler is often scuffed leading to imprecision. Sometimes we can estimate an extra digit of accuracy by “eyeballing” the next digit of a measurement. We want to insure that we include this digit when we report data. Figure 1.3 illustrates these ideas.

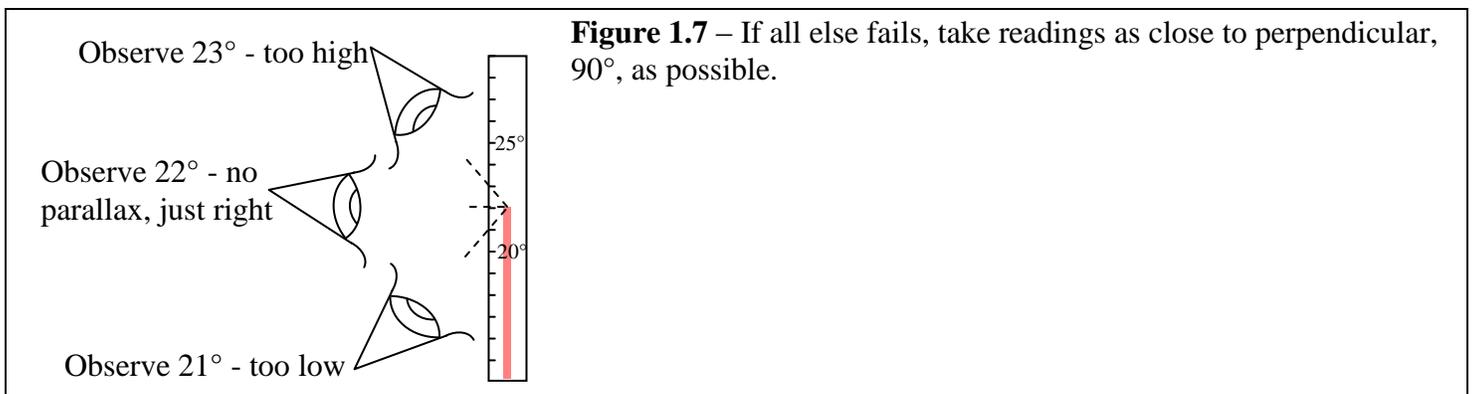
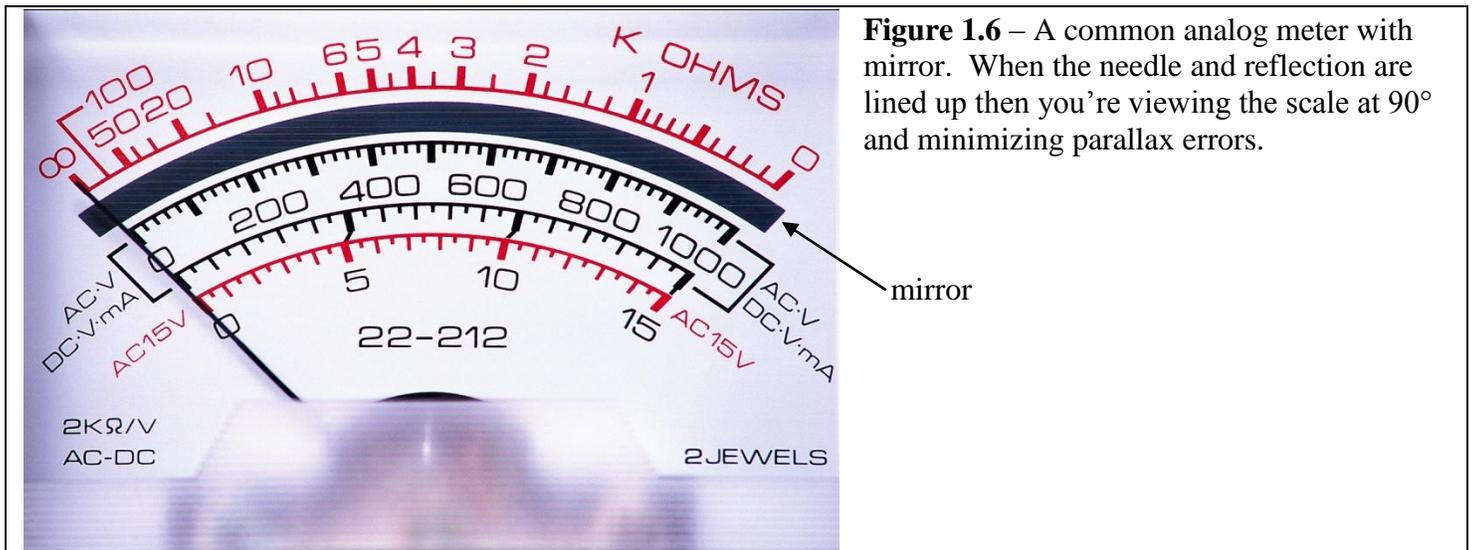
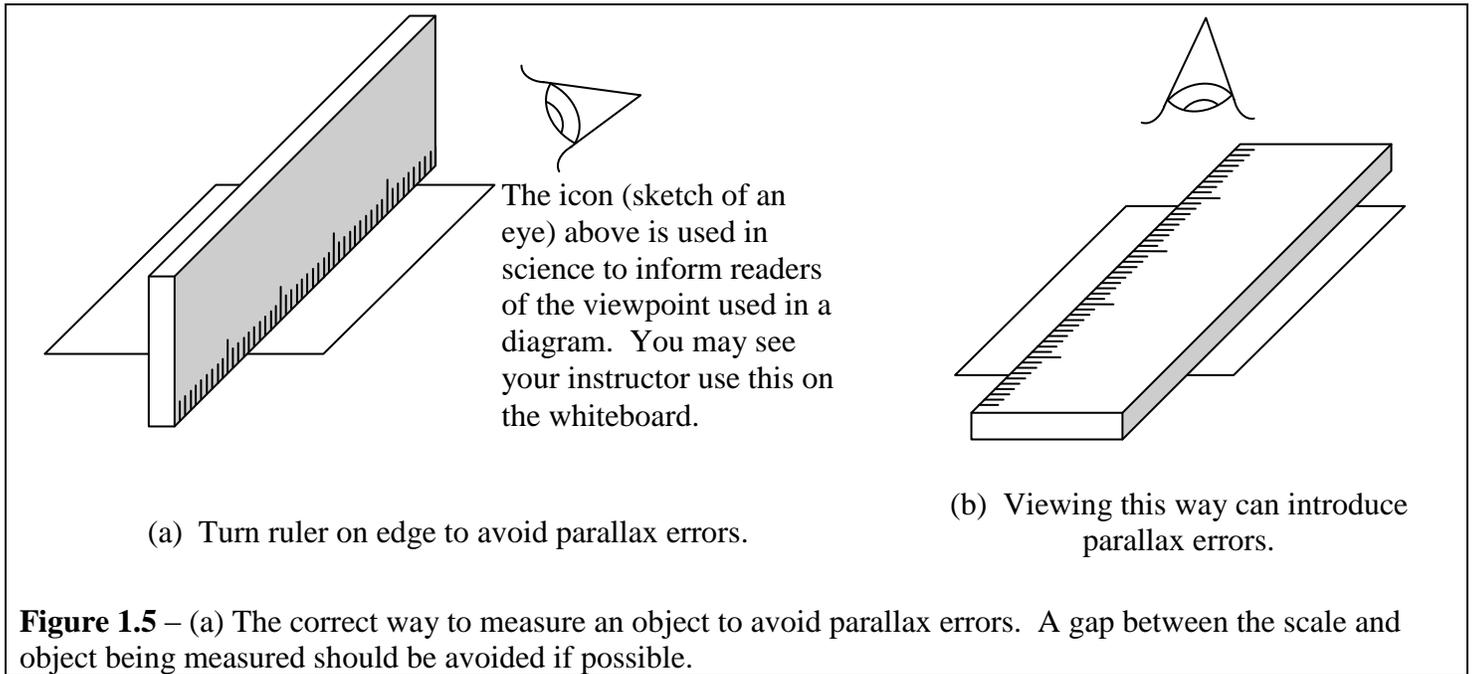


It’s important to minimize parallax errors also. There are a few ways to do this. The first is to place the marks on a ruler directly against the object being measures. Some rulers are beveled to accomplish this. Figure 1.4 shows this.



One way to put the scale next to the object measured is to turn the ruler 90° as shown in Figure 1.5. However it’s not always possible to do this. In common analog meters (Figure 1.6) the needle is displaced from the scale. However engineers devised a method to avoid parallax errors. They put a mirror in the scale. If the reflection of the needle is hidden by the needle, then you know you’re viewing the needle at exactly 90° and minimizing

parallax errors. If all else fails, just be careful to view the measurement and scale at 90° . A thermometer is a common device where the scale is unavoidably displaced from the inner liquid column. View it at 90° as shown in Figure 1.7 to minimize parallax errors.



H. Mean and standard deviation

Statistics are methods to comprehend random error, however, before we get started, we need to remember we cannot blindly apply statistics. A good example is pennies made prior to 1982 and pennies made after. They look identical, but before 1982 they were nearly pure copper (3.1 to 3.2 gm), but after they were 97.5% zinc clad with copper (2.5 gm). Blind application of statistics would give us nonsense. It's a good idea, therefore, to make a *histogram* of any data. A histogram is a plot along the vertical (y) axis of the number of times a data point occurs (frequency) with its value along the horizontal (x) axis. An example follows.

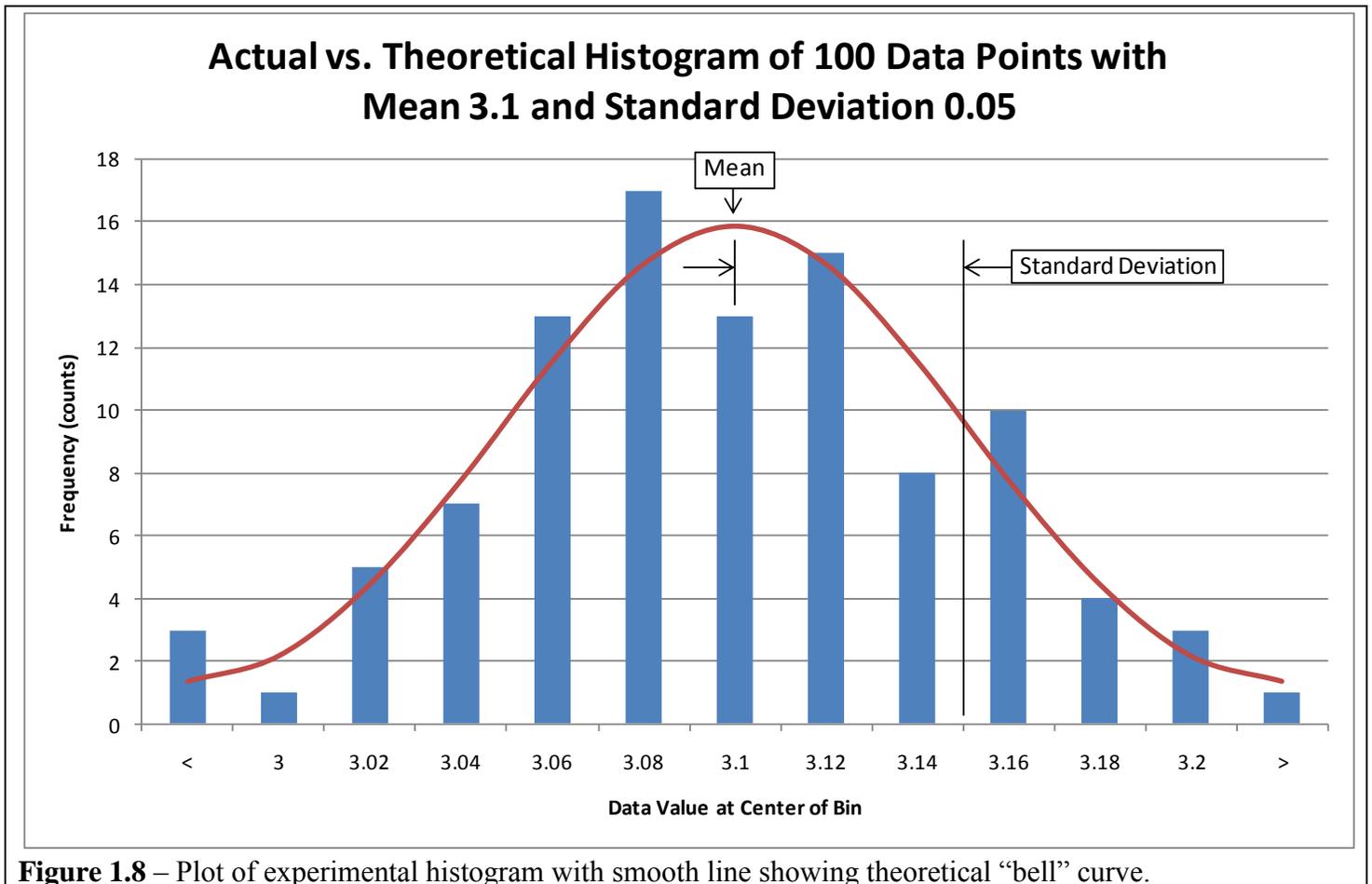


Figure 1.8 – Plot of experimental histogram with smooth line showing theoretical “bell” curve.

The smooth curve is bell shaped and actual data are the bars. The bars and curve don't match up exactly, and they should not, but they're pretty close. If our data looks like this, then we have good confidence in our information. If not, we need to think about our experiment and why our data may not follow this curve.

We don't expect you to calculate the smooth curve, but we do expect you to create a histogram. The numbers along the x-axis refer to the center. For example for the bar above the “3” if a data point was above 2.99 and less than 3.01 then the frequency incremented by 1 (the height of the bar increased one unit). The bar above the “<” sign is for data less than 2.99 and the bar above the “>” sign is for data greater than 3.3. While the mean was 3.1, the median was 3.09. The median is the center of the data and the mean should be pretty close to center. Half the data points are above the median and half below. Both the mean and median are measures of the central tendency, that is, the center of the data. The median, though, is a robust statistic. If you have data points that are way off, the mean may change dramatically, but not the median. If the mean and median are far apart then something is wrong (like outliers) and needs to be fixed. The statistics in your calculator and computer (mean, standard deviation, linear regression) are prone to be skewed by outliers. Your instructor is a good resource to help understand what may be happening in your experiment if the data is “odd.” That

calculators and computers can be skewed greatly is why you DO NOT BLINDLY PLUG NUMBERS INTO YOUR CALCULATOR OR COMPUTER and expect valid results.

We've already described how to find the median, but we haven't described how to find the mean and standard deviation. If you're confident your data is valid, by plotting a histogram for example, then the mean and standard are useful for a measure of central tendency (mean) and a measure of the spread (standard deviation). From Figure 1.8 you can observe that standard deviation measure the "spread" of the bell curve. Some data will be below the mean and some above – standard deviation (sort of) measures how much above and how much below the mean the data can be. This should be evident from inspecting Figure 1.8.

So far we've been attempting to provide an intuitive grasp of the concepts on mean and standard deviation. One more little thing before diving into the math. The most common practice is to put a line over the variable to indicate the mean:

$$\text{mean} = \bar{x} = \frac{1}{N} \left(\sum_{i=1}^N x_i \right) \quad \text{Equation 1.5}$$

N is the number of data points "i" is an index for each individual data point. It is mathematically incorrect to use μ for mean although you see it used this way often. Mathematicians reserve the symbol μ for the *population* mean (mean of everything) while \bar{x} is the *sample* mean (mean of sample drawn from everything). The subtle difference is buried in the mathematics of statistics which you will learn if you take a statistics class. Science and engineering usually deals with sample statistics so don't use μ – use \bar{x} instead.

Let's do an example – find the mean of: 3.09, 3.08, 3.16, 3.22, and 3.14. $N = 5$ and $\left(\sum_{i=1}^N x_i \right) = 3.09 + 3.08 + 3.16 + 3.22 + 3.14 = 15.69$ leading to the mean of 3.138. Make sure you can do this on your calculator. Read your calculator instructions if necessary.

The sample standard deviation is usually referred to as "s," however like the mean it is often referred to as σ (which refers to population standard deviation). Use "s" for sample standard deviation and not σ .

$$s = \sqrt{\frac{\sum_{i=1}^N d_i^2}{N-1}} = \sqrt{\frac{\sum_{i=1}^N (x_i - \bar{x})^2}{N-1}} \quad \text{Equation 1.5}$$

Note that $|d_i| = |x_i - \bar{x}|$ and is called the individual deviations. Refer to the example that follows in Table T1.1.

There's a math trick we can use: $\sum_{i=1}^N (x_i - \bar{x})^2 = \sum_{i=1}^N (x_i^2 - 2x_i\bar{x} + \bar{x}^2)$. Understand that \bar{x} is a common factor, therefore: $\sum_{i=1}^N (x_i - \bar{x})^2 = \sum_{i=1}^N (x_i^2) - 2\bar{x} \sum_{i=1}^N x_i + N\bar{x}^2$. Using Equation 1.5 we finally arrive at: $\sum_{i=1}^N (x_i - \bar{x})^2 = \sum_{i=1}^N (x_i^2) - N\bar{x}^2$. We can now rewrite standard deviation as:

$$s = \sqrt{\frac{\sum_{i=1}^N (x_i^2) - N\bar{x}^2}{N-1}} \quad \text{Equation 1.6}$$

Equations 1.5 and 1.6 are completely equivalent. Let's see how this works using our 5 numbers from before (and we already know the mean).

Table 1T.1 – How to calculate mean and standard deviation

	Data Value	Calculate s From Equation 1.5			Calculate s From Equation 1.6	
		Absolute Deviation, $ d_i = x_i - \bar{x} $	Deviation Squared, $ d_i ^2$		x_i^2	
Mean Calculation	3.09	0.048	0.002304	s calculation	9.5481	s calculation
	3.08	0.058	0.003364		9.4864	
	3.16	0.022	0.000484		9.9856	
	3.22	0.082	0.006724		10.3684	
	3.14	0.002	0.000004		9.8596	
$\text{Sum} = \sum_{i=1}^N x_i$	15.69		0.01288	$\text{Sum} = \sum_{i=1}^N d_i^2$	49.2481	$\text{Sum} = \sum_{i=1}^N x_i^2$
N	5		4	$N - 1$	0.01288	$\sum_{i=1}^N (x_i^2) - N\bar{x}^2$
$\bar{x} = \text{Sum}/N$	3.138		0.00322	s^2	4	$N - 1$
			0.056745044	s	0.00322	s^2
					0.056745044	s

You'll notice that the calculation for s yields identical results whether you use Equation 1.5 or 1.6. Just a couple more things to conclude our discussion of mean and standard deviation.

Percent Deviation is the ratio of standard deviation to the mean expressed as a percent. The formula is:

$$\%Dev = \left| \frac{s}{\bar{x}} \right| \text{ expressed as a percent} \quad \text{Equation 1.7}$$

While percent deviation measures how precise your measurements are as a percentage, *Percent Error* is how close you are to an accepted value and therefore indicates accuracy. Note the short “a” sound in “error”, “accepted”, and “accurate”. This is a mnemonic device to remember “error compares to an accepted value and measures accuracy.” The formula for percent error is:

$$\%Err = \left| \frac{\bar{x} - A}{A} \right| \text{ expressed as a percent where } A \text{ is the accepted value} \quad \text{Equation 1.8}$$

We have one more called Percent Difference. It's a little like %Dev, but uses the difference between the minimum and maximum measurement instead of s and should be several times larger than %Dev. The formula is:

$$\%Diff = \left| \frac{Max - Min}{\bar{x}} \right| \text{ expressed as a percent} \quad \text{Equation 1.9}$$

I. Graphing

You've already seen an example of one graph – the histogram. You should also have been taught graphing on your previous math classes. So this information should merely be review. Before we go over the rules, let's be clear, **DO NOT USE A COMPUTER OR CALCULATOR PROGRAM** unless you've taken a class in that program, for example Excel, and can apply the following rules. For the vast majority of students this means you'll graph by hand using graph paper. You should become very fluent graphing by hand before you start

using a computer or calculator program. There is an old saying about computers, “garbage in, garbage out.” In other words, using a computer or calculator doesn’t give you better answers. You have to learn to be smart about the information you give a computer so you’re not giving it garbage. And to learn to be smart you have to start by graphing by hand. There’s a later experiment, using the Geiger counter, where you must graph accurately by hand or you risk damaging a very expensive piece of equipment. Graphing by hand is like walking before you can run. You must become fluent in this skill.

Let’s just go through the rules for graphing:

- 1) Title your graph
- 2) Title both the vertical (y) and horizontal (x) axes
- 3) Include dimensions in the titles on both the vertical (y) and horizontal (x) axes
- 4) Choose a point that represents zero for both the horizontal and vertical axes (it’s called the origin)
- 5) Use the whole page – go big, not small, with your graphs
- 6) Choose to increment each major division (recall there are 5 or 10 minor divisions per major division) by an easy to use number like 2, 5, 10, 20, 50, 100, 200, 500, 1000 etc. Or, going down in values, each major division should be 1, 0.5, 0.2, 0.1, 0.05, etc. Don’t use a number like 3 – it’s hard to figure out the values for the minor divisions.
- 7) Make sure your minimum and maximum values will fit on the page. If not drop your scale to the next lower value according to the prior rule.
- 8) Optional, but very good idea to put names of lab partners and date on the graph
- 9) If you do a best fit (a later topic) put the equation of the best fit on the graph.

Let’s view an example of a poor graph and a good graph. We will plot measured distance vs. time for a car traveling at 27 m/s (about 60 mph) with random errors from, for example, human error in starting and stopping a stopwatch. First note, when we say “distance vs. time” we are informing you to plot distance on the vertical axis and time on the horizontal. Plot the “vertical variable versus horizontal variable”. Distance is the *dependent* variable and time is the *independent* variable. “X is independent.”

Here’s the information to plot:

Time (sec)	Distance (m)
0	0
1.48	20
1.51	40
3.08	60
3.54	80
3.99	100
4.79	120
5.75	140
6.39	160
7.43	180
8.23	200

The first column is our horizontal or x variable just like the first number in an ordered pair. Indeed, we plot the following ordered pairs: (0, 0), (1.48, 20), (1.51, 40), (3.08, 60), (3.54, 80), (3.99, 100), (4.79, 120), (5.75, 140), (6.39, 160), (7.43, 180), (8.23, 200). First a poor example.

Figure 1.9 – Example of a poor graph

Example of Poor Graph:

1. Too Small
2. No origin
3. Vertical and horizontal switched
4. Uneven scale. On vertical axis 3 divisions from 1 to 3 ($2/3$ unit per division) on vertical axis. 5 divisions from 3 to 6 ($3/5$ units per division). Horizontal scale 16 units per division from 20 to 100 and 20 units per division from 100 to 200.
5. No axis units
6. No axis titles
7. Graph not titled
8. Odd number ($2/3$, $3/5$, etc.) of units per division.
9. No best fit line (would be meaningless anyway).
10. Labels don't fit on page

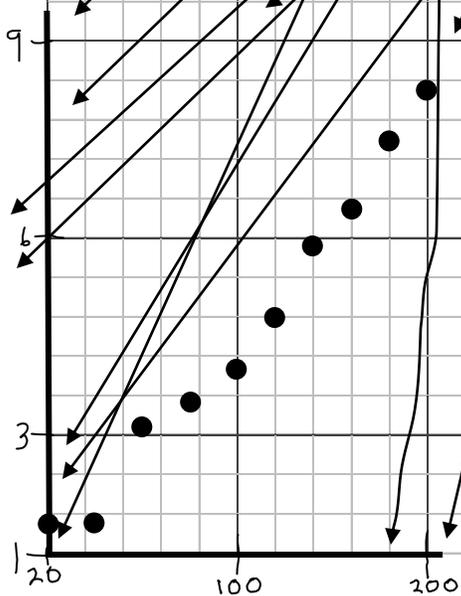
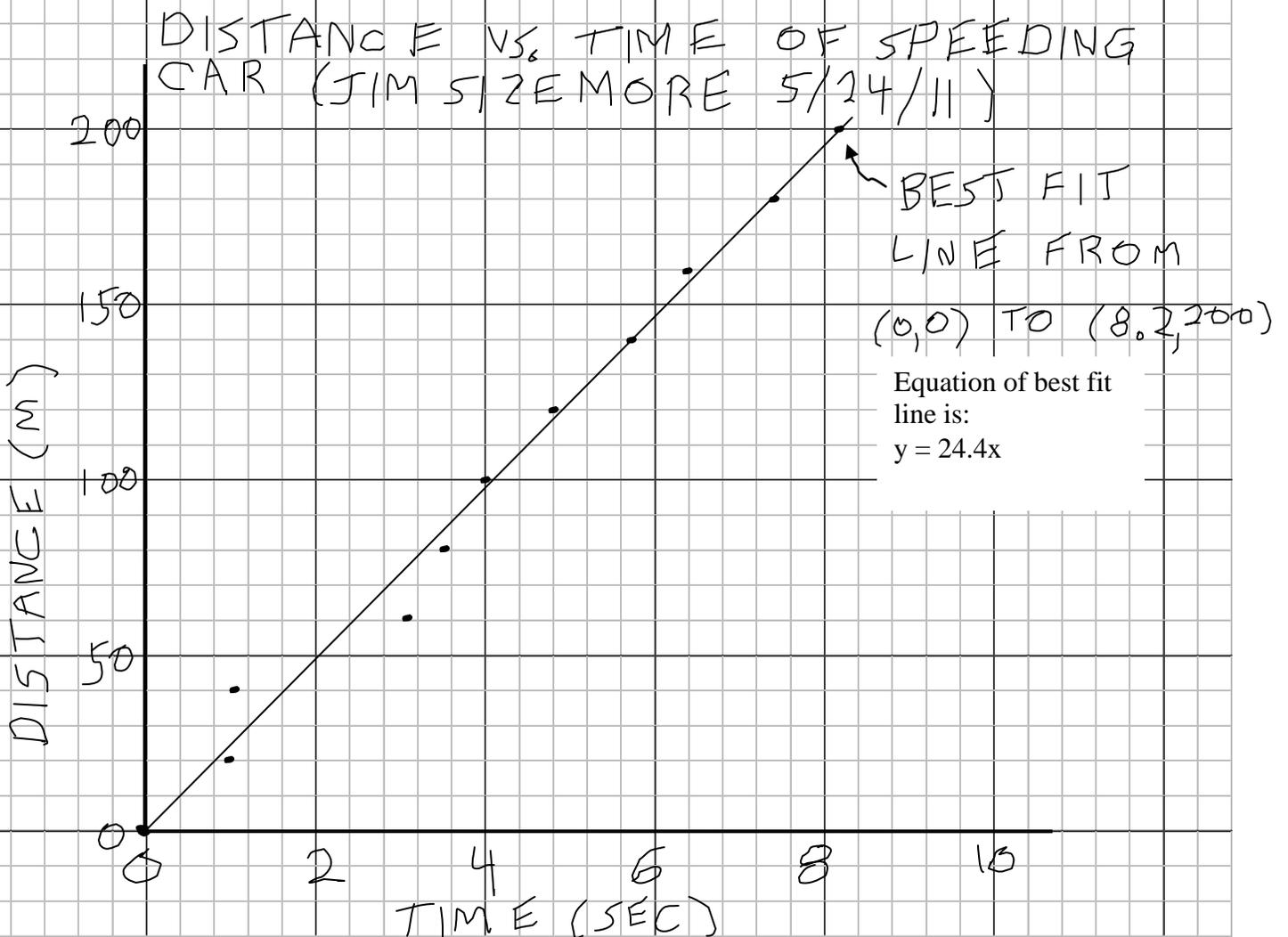


Figure 1.10 – Example of a good graph



J. Best Fit Straight Line

Take a look at Figure 1.10, the example of a good graph, again. On it we fit a straight line. Sometimes data is curved and we'll deal with that in the next section. But in the case of Figure 1.10 the data, in theory, fits a straight line. But not all the data fits on the line due to random errors. In this example, 3 points fit on the line, 3 points are below and 4 points are above the line. This is about as good as it gets. The best fit line goes through the "middle." About an equal amount of data is above the line as below the line. Also note that, since at zero time, the car travels zero distance the line MUST pass through (0,0). Often this will be the case. The other case is: $y = mx + b$. Because the line passes through (0,0) the intercept $b = 0$. For other problems b might not equal zero. Be sure you understand if your experiment has a non-zero intercept or not.

The slope of the line is 24.4 (intercept is zero) and, since the y axis is in m and the x axis is seconds, the speed of the car is 24.4 m/s. That's our measurement. Remember, previously, we said the car was traveling 27 m/s. We got kind of close, but not the exact value. This is normal and expected. What's the percent error for our measurement? Answer: 9.7%.

Why don't we just use a computer to figure the "best fit" line? Mostly because the computer blindly applies an algorithm that may, or might not, be appropriate. Least squares statistics (the kind most computers use) are highly prone to leveraging due to outliers. It takes a lot of study in statistics to understand when least squares statistics fails and what to do about it. This study is way beyond the scope to this course. For now, plot the best fit line by hand. For your information, though, our speeding car data is quite good and the least square algorithm results in a speed of 24.2 m/sec which is actually a little worse than figuring the best fit line by hand.

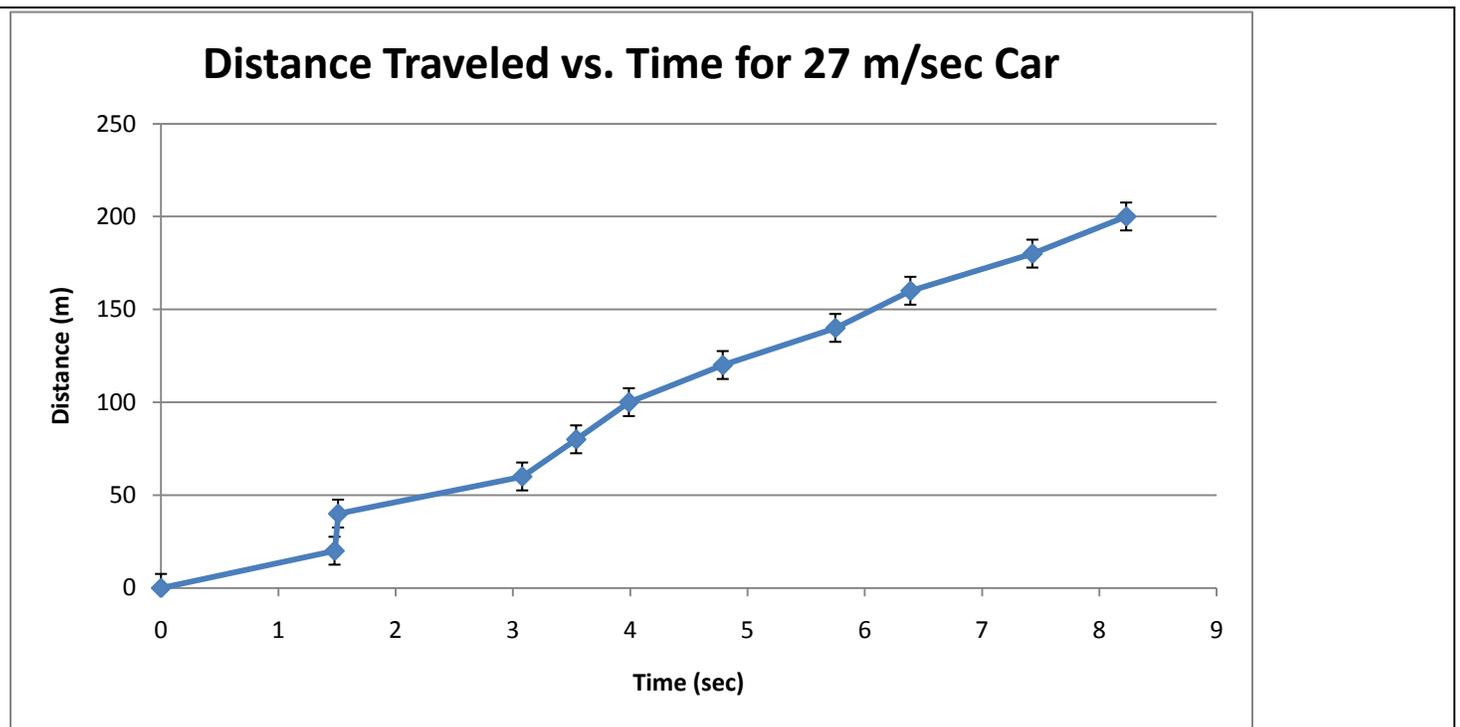


Figure 1.11 – Graph of our speeding car data with error bars. We add one standard deviation to the y data to get the upper horizontal bar and subtract one standard deviation from the y data to get the lower horizontal bar. Look near 1.5 sec. We have two data points with nearly equal x values, but the y values are more than two standard deviations apart. Ideally 68% of data will fall between the data less a standard deviation and the data plus the standard deviation. These two data points are part of the 32% that can have differences greater than a standard deviation. It doesn't happen the majority of time, but 32% is a significant minority.

If we know what the standard deviation is (if we repeat the measurement many times) we can put error bars on our data points. We presume the horizontal variable is exact (which may or may not be the case). The algorithms to handle information if the horizontal variable is inexact are very complicated requiring the study of advanced statistics beyond the scope of this course. Figure 1.11 shown previously shows speeding car data with error bars of 7.5 m.

K. Transforming Data

We're almost done with the fundamentals of calculation, measurement, and graphing. We have one last topic and that has to do with data that does not fit a straight line. Often, however, we can square, take the square root, exponentiate, take the logarithm, etc. of one or more variable to get a straight line equations. This is called *transforming* variables. Let's take a look at a few examples to see how this works.

One of the first equations you'll be dealing with in the course concerns uniform accelerations and is an equation of distance (x) given time (t). That equation is:

$$x = \frac{1}{2}at^2 \qquad \text{Equation 1.10}$$

Instead of plotting x vs. t , we plot x vs. t^2 . Consider the x vs. t data (0, 0), (1, 2.1), (2, 20.7), (3, 44.9), (4, 81), (5, 124.9), (6, 178.9), (7, 238.4), (8, 319.9), (9, 399), (10, 488.9). Let's make Table T1.1.2 as follows:

Table 1T.2 – Table of x vs. t data with t^2 calculated.

Time, t (sec)	Distance, x (m)	t^2 (sec ²)
0	0	0
1	2.1	1
2	20.7	4
3	44.9	9
4	81	16
5	124.9	25
6	178.9	36
7	238.4	49
8	319.9	64
9	399	81
10	488.9	100

The trick is, instead of plotting Column 2 (distance x) vs. Column 1 (time t), we plot Column 2 (distance x) vs. Column 3 (time squared or t^2). Let's try it – be sure to use the best graphing technique you learned previously. The graph is shown in Figure 1.12. The accepted value of acceleration is 9.8 m/sec² and Figure 1.12 shows a value of 9.8458 m/sec². Pretty close.

Here's a few more examples of how to rearrange data. For:

$$x = \frac{1}{2}at^2 + x_0$$

Plot x vs. t^2 , but this time you'll have an intercept in addition to a slope.

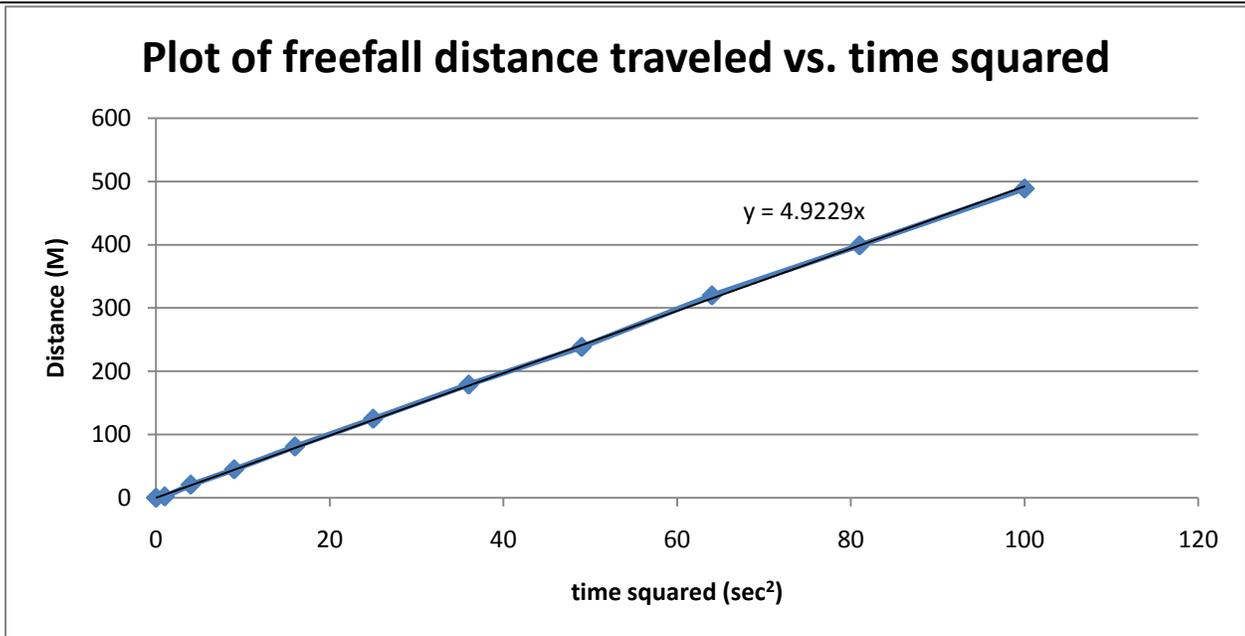


Figure 1.12 -- Graph of Table 1T.2 data. The equation of the line is shown and the slope reported is the measured value of $\frac{1}{2} a$ per Equation 1.10. This means $a = 2 * \text{slope} = 9.8458 \text{ m/sec}^2$. This is pretty close to the accepted value of 9.8 m/sec^2 .

If you have an equation like this:

$$N = N_o \left(\frac{1}{2}\right)^{\frac{t}{t_h}}$$

Take the log of both sides to get:

$$\log(N) = \log(N_o) + \left(\frac{t}{t_h}\right) \log\left(\frac{1}{2}\right)$$

So plot the $\log(N)$ vs. t and the slope will be $\log\left(\frac{1}{2}\right) / t_h$ and the intercept $\log(N_o)$. For an equation like this:

$$V = V_o \left(1 - e^{-\frac{t}{RC}}\right)$$

Rearrange to obtain:

$$\frac{V_o - V}{V_o} = e^{-\frac{t}{RC}}$$

Take the natural log of both side to get:

$$\ln\left(\frac{V_o - V}{V_o}\right) = -\frac{t}{RC}$$

So if you plot $\ln\left(\frac{V_o - V}{V_o}\right)$ vs. t you'll get a slope of $-\frac{1}{RC}$ and the theory predicts zero intercept. So your best fit line better intercept zero. One more example:

$$d_b = 10 \log\left(\frac{I}{I_o}\right)$$

Transform the equation by a little rearrangement and exponentiating both sides to obtain:

$$10^{\left(\frac{d_b}{10}\right)} = \frac{I}{I_o}$$

In this case then, plot $10^{\left(\frac{d_b}{10}\right)}$ vs. I to get a slope of $\frac{1}{I_o}$ and intercept of zero. These are just a few examples.

There are a plethora of problems requiring some data transformation to make sense of information. Your job is to understand the problem, recast it, and then transform both sides to obtain a linear equation. After this course an advanced statistics can help you fit equations that are nonlinear also. However the best choice, if possible, is to rearrange equations to obtain a linear equation.

Conclusion

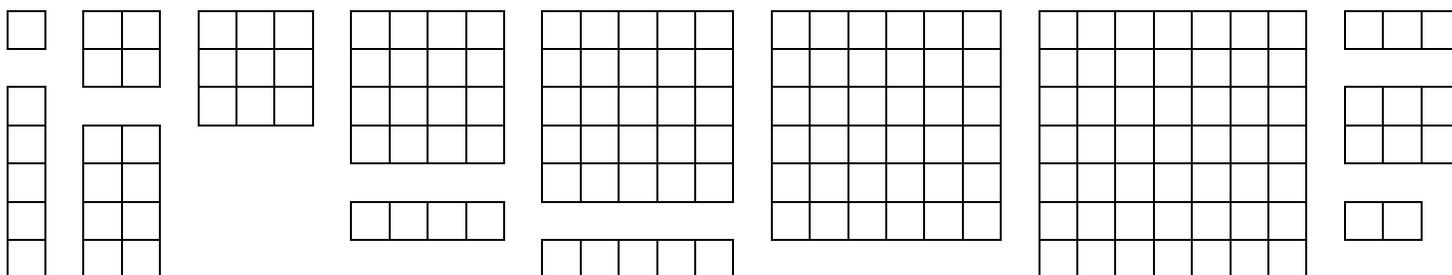
We've discussed many techniques to report, graph, and analyze data. This basic understanding will be important to apply to the remainder of experiments you will perform this term. Many students feel they can learn something and then forget it when we go to the next chapter. That won't work in this class especially for this lab. You will be applying this information the remainder of this term.

Procedure

1. Perform the scientific notation calculations in Problems 1 to 10. Refer to the Theory Section, Part A.
2. Perform the units conversion in Problems 11 to 13. Refer to Theory Section, Part C.
3. Solve and perform dimensional analysis in Problems 14 and 15. Refer to Theory Section, Part D.
4. Solve Problem 16 using the SOLVE method. Refer to Theory Section, Part E.
5. Round these numbers to 3, 4, and 5 sig figs and fill out Table 1.1: 0.164655759, 0.822001353, 634.6495394, 2.50768E-18, 8.85658E+12. Refer to Theory Section, Part F.
6. Solve Problems 17 to 19. Refer to Theory Section, Parts G through J.
7. Following are a series of rectangles. Record the measured diagonal, h ; the ratio of width to length, r ; and the area, a . Count squares to get a and r . The formula relating these three variables is:

$$a^2(1 + r^2) = h^2$$

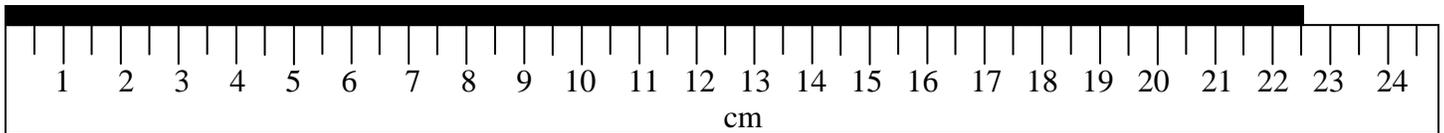
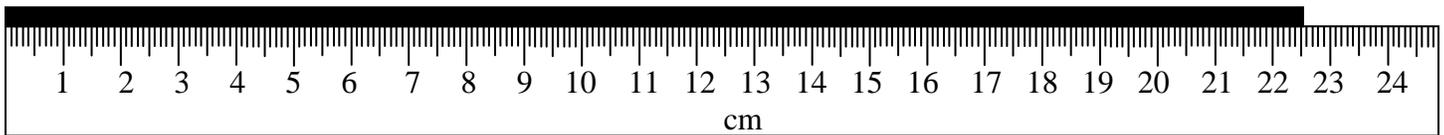
Find the linear equation of a vs. $f(r, h)$ (Hint: you want to rearrange to get $a = \text{function}(h, r)$). Create the data table, this linear equation, graph and analyze, and attach separate page to lab report.



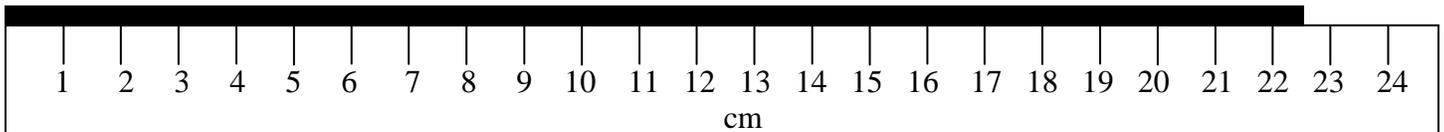
Post-Lab Questions – Answer on a separate page and attach to your lab report

1. Which of the following rulers will give you the most precise measurement? Least precise? Explain.
Give the measurement of the black object to the correct number of significant digits for all three rulers.

Ruler 1



Ruler 2



Ruler 3

2. In Problem 19 (Lab Report Section), was the same timer used to measure all times? Explain.
3. The speed of light, c , may be measured using a microwave oven at 2450 MHz frequency, f , and placing a glass dish with marshmallow topping, eggs whites, or something else that indicates where the hot spots are. It is expected that hot spots will occur at anti-node $\lambda/2$ apart. The equation is $c = \lambda f$ where λ is in m and f is in Hz. The accepted value of $c = 3 \times 10^8$ m/sec. If antinodes are measured at 6 cm apart, what is the measured speed of light? Is this measurement precise? Is this measurement accurate? Explain.
4. What statistic would you use to indicate precision? Accuracy? Explain.
5. If you were the first person to measure c per Question 3, how would you describe and what statistic would you use to explain the experimental uncertainty. Hint: The least count for this problem is 1 cm.
6. What is the average deviation for Problem 18 in the Lab Report Section. Is it zero? Explain.
7. How would you measure the period of a pendulum precise to 0.01 sec using a stopwatch? How would you measure it accurate to 0.001 sec using a stopwatch?

Lab 3 Report: Names _____

1) Fill in the following table:

Convert these numbers into scientific notation	Convert these numbers into standard notation
265,000	5.49×10^{-17}
78,100,000	6.72×10^{10}
48,900,000,000,000	4.44×10^{-8}
0.00000000000000529	8.98×10^{-11}
0.00000000000000258	8.42×10^{20}

2) What is $10^{-4}/10^{-4}$? Do not use your calculator. _____

3) Now use your calculator to find: $10^{-4}/10^{-4}$ _____

4) Did you get the same answer to Problems 1 and 2? Why or why not? _____

5) Attempt Problem 2 until you get the correct answer. What is the correct answer? _____

6) What is $10^{-4} \times 10^4$? Do not use your calculator. _____

7) Now use your calculator to find: $10^{-4} \times 10^4$ _____

8) Did you get the same answer to Problems 5 and 6? Why or why not? _____

9) Attempt Problem 6 until you get the correct answer. What is the correct answer? _____

10) What is $\frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})}{(6.38 \times 10^6)^2}$? _____

11) Convert 36 in to m where 2.54 cm = 1 in. _____

12) How many km is 1 mile? 2.54 cm = 1 in, 12 in = 1 ft, 5280 ft = 1 mile. _____

13) What is the mass of a 200 lb person? Mass of 1 lb = 0.45359237 kg _____

14) Find the acceleration of gravity, g , in $\frac{m}{sec^2}$ using the equation $F = mg$ if $F = 1.47$ N and $m = 100$ gm _____

15) The acceleration of gravity, g , is often found using pendulums and the equation for the period of a pendulum, T : $T = 2\pi \sqrt{\frac{l}{g}}$. Find g in $\frac{m}{sec^2}$ for $l = 100$ cm and $T = 2.01$ sec. Derive a formula for g before performing calculations. _____

16) At sunrise a frog falls into a well 5 m deep. During the day it hops up 3 m and at night falls back 2 m. How long will it take the frog to get out of the well? This seems like a simple problem, but it's essential to use the SOLVE method to find the solution. Show your work on a separate page _____

Table 1.1 – Rounding to Specific Number of Sig Figs.

Sig Figs – write original number here →					
3					
4					
5					

17) What is $3 \times 7 \times 11$ and how precise is the answer? _____

18) Find the mean, standard deviation, %Dev, %Err (accepted value is 9.8), and %Diff of 8.54, 9.97, 8.61, 9.17, and 9.63. Also comment if you think this data is precise and/or accurate? _____

19) Plot the following distance (in m) vs. time (in sec) data and find the best fit line including non-zero intercept: (0, 2.73), (10, 98.5), (20, 189.1), (30, 292.1), (40, 384.4), (50, 498.1), (60, 605.5) _____