Labs: Trigonometry Based Physics Part I

EXP 3 – Lab

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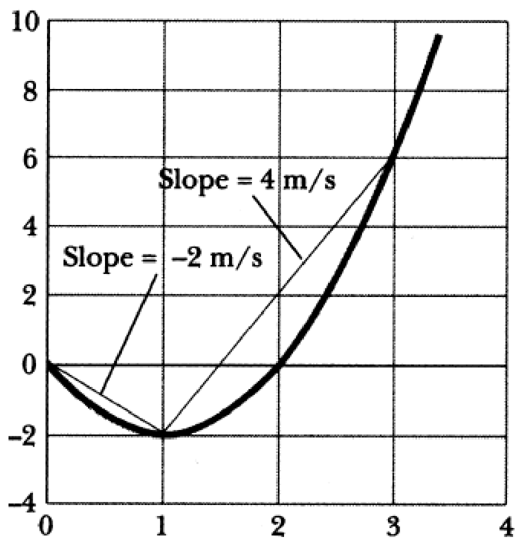
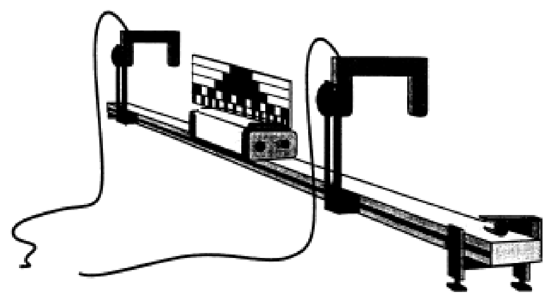
Velocity & Acceleration





Purpose

You will investigate the motion of objects moving with constant acceleration.

Equipment

- Track – one of the following options:
  - Air track
  - Pasco track
  - Hot Wheels or other inclined plane track
- Stopwatch (or multiple stopwatches)
- Scissor jack, blocks, pieces of 2x4, or other blocks to raise track
- Free fall apparatus – one of the following options
  - Pasco free fall apparatus that depends on conductivity of object to trigger timer start
  - Spark timer free fall apparatus
  - This lab may be performed by simply dropping masses and using a stopwatch (with some loss of accuracy).
- Three objects of different masses – objects must be conducting if using an apparatus that relies on conducting objects such as the Pasco free fall apparatus
- Meter stick or metric tape measure

Introduction

At first glance, it seems obvious from our everyday experience that here on Earth (1) inanimate objects at rest do not move of their own accord, and (2) if you put them into motion they soon return to rest. Based on such observations, Aristotle developed a theory that any object’s natural state is to be at rest and that being in motion is an unnatural (temporary) state. His theory was accepted by the world's philosophers for over two thousand years.

But Galileo and Newton asked questions about the motion of objects that could not be answered by Aristotle's theory. In seeking answers to their questions, they ultimately produced a new theory of motion that applied not only to all objects here on Earth, but to everything in the heavens as well. This theory has since been developed more fully and is now accepted by educated people all over the world.

Our current view of motion is that, when all interactions between an object and its environment are considered, any object in constant-velocity motion is just as much in a natural state as is an object at rest. For an isolated object (an object that does not interact with its environment through friction or air drag), no effort or force is required to maintain its constant motion, just as no effort is required to keep it at rest. We can only imagine such an isolated object, of course, because in real life there is always some small friction or drag acting on moving objects on Earth.

The tendency of any object, then, is to remain in whatever state of constant motion it is already in (including constant zero motion – that is, being at rest) as long as no external force is acting on it. This tendency to avoid any change in motion is due to the object's inertia. Because of an object's inertia, a net force must be exerted on it to make its state of motion change – from rest to moving, from moving to rest, from moving at one speed to moving at another, or from moving in one direction to moving in another.

The relationships between an object's position, its speed, its acceleration (rate of change of speed), and time are investigated in a branch of physics called kinematics. The simplest case, the one you will examine in this experiment, is motion in a straight line – called one-dimension motion.

In this experiment, you will:

1) Become familiar with some basic measurements of motion.
2) Learn to make and read graphs of position, speed, and acceleration vs. time.
3) Learn the distinction between average and instantaneous speed and acceleration.
4) Investigate one-dimensional motion at constant acceleration.

Dynamics of Motion

For an object in motion, its position is a function of time [$x = f(t)$]. You can completely describe the motion of any object at any time (in the past, the present, or the future) by describing this function; i.e., by stating the quantitative relationship between its position, speed, and time.

Although a moving object's position varies with time, either its speed or acceleration may be either variable (can vary with time) or is constant. Furthermore, since any object's position is relative to some coordinate system, its motion is also relative – what appears as motion to one observer may appear as rest to another observer if the two observers are moving with respect to one another.

Speed (Velocity)

If an object's position varies with time, we say it is in motion. If the position is constant in time, the object is at rest. We define an object's displacement as the quantity of change of its position, and its speed as the rate of change of its position. We write the relationship between speed (v), position (x), and time (t) symbolically as

$$\bar{v} = \frac{\Delta x}{\Delta t} \qquad \textbf{\textit{Equation 3.1}}$$

where $\bar{v}$ is the object's *average* velocity, Δx is its displacement and Δt is the time interval during which the position changed. Refer to the following figure:

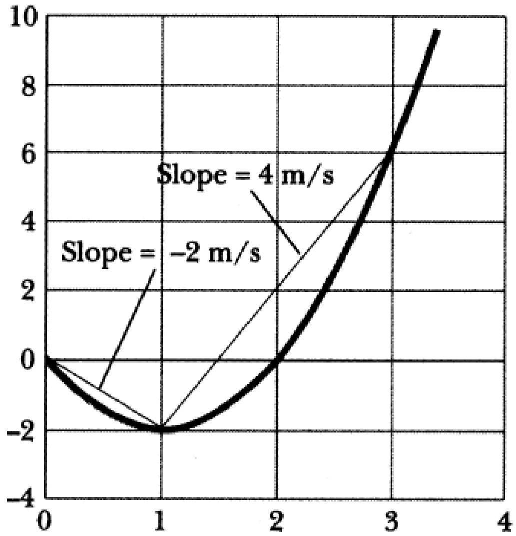


Figure 3.1 – Graph of position vs. time. Scale is meters on the vertical axis and seconds on the horizontal axis. The straight lines show the average speed between time zero and t = 1 sec and the time interval between t = 1 sec. and t = 3 sec.

The measurement unit for speed is simply the ratio of distance units to time units. It can be miles/hr, meters/min, feet/sec, furlongs/fortnight, or whatever.

But this simple relationship doesn't give us all the details of the object's motion. The most obvious question left unanswered is, "Did the object's velocity vary during the time interval Δt, or did it remain constant?" If the velocity remained constant, then you know its value was $\bar{v}$ at every instant of time during the interval. But how do you find the value of velocity at any instant if it varied during the time interval? This question leads us to two concepts of velocity.

The first concept is called average velocity. It is the ratio of the object's displacement to the time interval during which it moved. Equation 2.1 expresses average velocity. For example, it's easy to determine how long a 200-mile trip will take if you know that your average velocity on it will be 50 miles per hour.

On the other hand, the highway police don't care what your average velocity is over the 200 mile trip. They want to know how fast you are driving at the instant they are observing you (when their radar beam strikes your car), so they can decide whether or not you are exceeding the speed limit at that particular point in time. The highway cop wants to know your *instantaneous* velocity.

Your car's speedometer indicates your instantaneous *speed* – your speed at every instant of time. Your trip odometer divided by total trip time is the average speed. The equation for instantaneous speed appears similar to the equation for average speed, except instantaneous speed is measured over very small time intervals. In math we use limits to express this idea as shown in Equation 2.2. This is all we need for now, but if you go on in math or physics you will study calculus to better handle varying acceleration or other, more complicated, scenarios such as air drag or aircraft flight, etc.

$$v(t) = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

Equation 3.2

Acceleration

Another question unanswered by these two equations for velocity is, "When the velocity is changing, how fast is it changing?" In other words, what is the rate of change of the velocity? Just as we call the rate of change of position velocity, we call the rate of change of velocity *acceleration*.

The measurement unit for acceleration is the ratio of velocity units to time units: miles/hour/sec, meters/min/sec and so forth. When Walter Cronkite covered space launches in the 1960 he would report the rocket is accelerating at "120 feet per second per second." This is somewhat confusing and misleading. Do you divide seconds by seconds and, therefore, wind up with units of feet for units of acceleration? Of course not! We take the fraction for units of velocity, m/sec for example, and divide by units of seconds. Remember the saying you learned in grade school: Dividing fractions, don't ask why, flip the second, and multiple. In an equation it looks like this:

$$\frac{\left(\frac{m}{s}\right)}{s} = \left(\frac{m}{s}\right)\left(\frac{1}{s}\right) = \frac{m}{s^2}$$ ***Equation 3.3***

Since m/s^2 is not confusing, scientists and engineers customarily, and nearly universally, report acceleration with the square of the time in the denominator (ft/sec^2, etc.).

As an aside, we can deduce the reasons why behind the saying, "Dividing fractions, don't ask why, flip the second, and multiple." Think about six divided by two. The answer is the number of groups of two in six. What about six divided by a half (½)? How many halves are in six? Twelve, of course. That's the same as flipping (taking the reciprocal) of the second number (half or ½) and multiplying. Flipping ½ becomes two and multiplying by six gives you twelve. Most of you know this by memory, rote, or practice, however one day most of you will need to explain this to your third grader or, if you go into teaching, you'll need to explain this to a lot of third graders.

Galileo determined that, in the absence of air drag, all freely-falling objects fall with the same constant acceleration, that is, their velocity increases steadily until they hit the ground, the floor, or whatever. This constant acceleration near the earth's surface is given its own symbol, g, and its value is $g = -9.80$ m/s^2. The negatively sign informs us that gravity acts downward since, customarily, up is positive and down is negative. Watch out though. Often when writers use "g" in an equation they mean the absolute value – not the signed value. You have to be smart enough to understand from the context, when you examine an equation, if the meaning is the absolute value or the signed value.

Because he didn't have the accurate interval timers we have today, Galileo cleverly used a timer that was always available, albeit not very stable – his heartbeat. Even so, he had to devise a method of slowing the descent of the object so its fall lasted more than one or two heartbeats. He realized that an object sliding down an (imaginary) frictionless inclined plane (a ramp) would also move with a constant acceleration, as would a ball rolling down the ramp. He also saw that the acceleration of such an object would be a fixed fraction of its free-fall acceleration, g, depending on the inclination angle of the ramp.

In your prelab you should have derived several equations. Write equations for distance an object falls given time and the case of zero initial velocity in your lab report.

If a moving object's acceleration varies (not a freely falling object, of course), you can write average and instantaneous values of its acceleration just as you wrote them for velocity. The average value of acceleration over the time interval Δt is:

$$\bar{a} = \frac{\Delta v}{\Delta t}$$ ***Equation 3.4***

Similar to velocity, in the limit as $\Delta t \rightarrow 0$ (meaning Δt becomes very small), the average acceleration approaches the instantaneous acceleration at the time t.

$$a(t) = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$ ***Equation 3.5***

<u>Graphing the Motion</u>

The relationships between position, x, time, t, velocity, v, and acceleration, a, can be described analytically, as in the previous equations, or as in the Figure 3.1 (shown previously). A graph of an object's position vs. time allows you to easily determine its average and instantaneous velocity. You can write the equation defining v as $\Delta x = vt$. If $x_0 = 0$, the equation becomes $x = vt$. A graph of x vs. t, such as shown, is a gold mine of information about the object's motion.

From this graph, we can see that the object's position when $t = 0$ is $x = 0$. Then it starts moving in the minus x direction and reaches $x = -2$m when $t = 1$s. After that, it reverses direction and is back at $x = 0$ when $t = 2$s. It continues moving in the positive x direction, reaching the position $x = 6$m when $t = 3$s.

By drawing chords between specific points on the curve, we can find the object's average velocity over a given time interval. In the example shown, the slope of the chord between any two points on the graph is equal to the average value of the velocity during that interval. For $0 < t < 1$s, the average velocity is – 2m/s, and for $1\text{s} < t < 3$s, the average velocity is + 4m/s.

As the time interval shrinks to a single point on the t axis, the chord becomes the tangent to the graph (the average velocity becomes the instantaneous velocity) at that instant in time.

If the velocity were constant, the slope of the graph would be constant (the graph would be straight). A positive slope indicates a positive value of v (the object's displacement, x, is increasing with time), a negative slope indicates a negative value of v, and a changing slope indicates a changing value of v.

Similarly, a graph of Δv vs. Δt will be a curve whose slope at any point is equal to the acceleration a. If a is constant, the slope of the graph will be constant (the graph will be straight). A positive slope indicates a positive value of a (the velocity is increasing with time), a negative slope indicates a negative value of a (the velocity is decreasing with time), and a changing slope indicates a changing value of a.

Even if motion is considered in two dimensions, graphs of position vs. time and velocity vs. time can be drawn for dimension individually. For example, the graphs below are for an object thrown straight up then falling back down again.

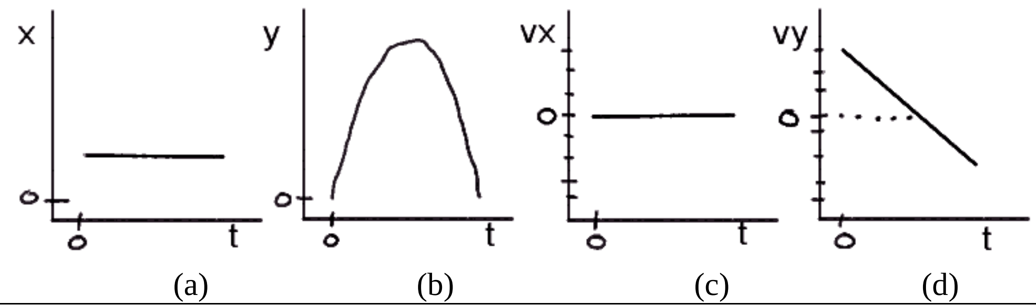


Figure 3.2 – Four sketches of position and velocity vs. time

Inclined Plane

As noted earlier, Galileo realized motion would slow if he used an inclined plane. In your pre-lab you should have derived an equation for acceleration down the plane (downhill is the positive direction) given |*g*| and θ. Write this equation and equations of *a* vs. *x, y, h* and |*g*|; and v_i, vs. d_i and t_i in your lab report.

Examine the figure below.

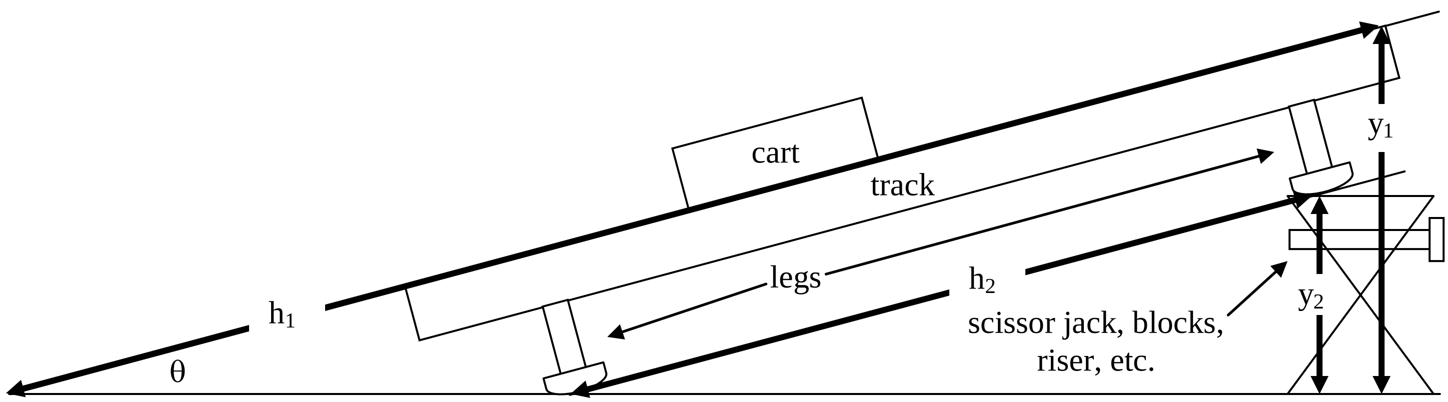


Figure 3.3 – Schematic diagram of inclined plane experiment

1) Will your results change if you measure y_1 and h_1 vs. y_2 and h_2? Will the angle be different if you measure y_1 and h_1 vs. y_2 and h_2? Will the acceleration down the plane change is you measure y_1 and h_1 vs. y_2 and h_2? Is $\frac{y_1}{h_1} = \frac{y_2}{h_2} = \frac{rise}{distance}$ a constant where rise is how much the cart rises and distance is the amount up or down along the plane that the cart travels? Do you have to measure the distance and rise of the cart or will these other ratios give you the same information? Explain thoroughly on separate page and attach to lab report referencing this item number.

Data Collection & Analysis

2) The experiment is divided into two part, free fall and inclined plane. In the free fall portion we will use three different masses dropped from three different heights with repeated measurements three times. The repetition of measurements allows us to calculate mean and standard deviation for each of the nine variations of mass and height. By comparing the means and standards between the nine groups we can ascertain if height or mass influences *g*, the acceleration of gravity.
3) The second portion of the experiment is on an inclined plane. In this case mass is constant, but distance and elevation are varied and we will repeat measurements. There are multiple ways in which data may be analyzed and we will use two of these – plotting distance, *d*, vs. time squared, t^2, and plotting instantaneous velocity, v_i, vs. time, *t*.

Part I: Free Fall

4) You will be dropping the three objects (of three different masses) three times each from three different heights for a total of 27 readings. If you're using an accurate electronic timer the three heights of 0.5 m, 1.0 m, and 1.5 m should work well. Even less distances (e.g., 0.3 m, 0.6 m, and 0.9 m) should work. Using stopwatches, to improve accuracy, I suggest you drop from higher heights (from a window for example). Use, for example, 2 m, 3.5 m, and 5 m (tape measures commonly end at 5 m). Review the Pre-lab (Item 7) for precisely what we mean by distance of fall. Practice first to accurately start the stopwatch when the object is released and stop the stopwatch when the object hits the floor or ground. Record your data in Table 3.1.
5) From Table 3.1 and using Equation 3.5 calculate individual values of $|g|$ and the mean, m_g, standard deviation, s_g, and percent error, *%Dev* of $|g|$ for each of the nine groups. Recall the accepted value of $|g| = 9.8\ \text{m/sec}^2$. Record this information in Table 3.2.
6) Can you confirm the hypothesis that $|g|$ is independent of mass? Is $|g|$ independent of distance of fall? Generally, if values are within +/- two standard deviations of each other we conclude that there is no difference. Is this true of the information in Table3.2? How well did your measurements match the accepted value? Generally we look at percent deviation, *%Dev*, to ascertain this. Explain thoroughly on separate page, include reference to this item (5), attach to lab report, and include a discussion of systematic or random errors.

Part II: Inclined Plane

7) Level the track. Generally the best way to do this is set the cart or air track glider on the track at rest and observe if it begins accelerating. Due to air currents, on an air track the glider may wiggle back and forth slightly, but as long as it does not begin moving consistently in one direction you are as close as you can get to level. You may also use a bubble level for any track. Do not move the glider on an air track without the air being on (it damages the track by scratching the scale on the side).
8) Mark the starting point and five equally spaced distances on the track. The last mark should be near the end of the track. The distances will depend on the track. For storage, generally, tracks are about 2.0 m and you need a little room for the beginning and the end. Thus, in most cases, marking the start, 0.3 m, 0.6 m, 0.9 m, 1.2 m, and 1.5 m should work well.
9) Launch the cart or glider at constant velocity. Good ways to achieve constant velocity is to compress a spring or rubber band (arranged like a sling shot), stretch the spring or rubber band a constant amount, and launch the cart using the spring or rubber band.
10) Measure the times required to reach each of the five marks, repeat each measurement three times, and record the data in Table 3.3 (below).
11) Raise the track. For stability, *raise the lighter end of the track AND insure the feet are completely on the blocks or scissor jack.* Failure to comply with this has resulted in apparently light weight tracks falling off tables with great force, injuring students, and damaging equipment. While the tracks appear light, this can lull students into a false sense of security.
12) It is recommended to raise it only a few degrees at a time and keep the maximum slope between 5° and 10°. Think about what this implies regarding the heights to raise the track. This time you will release the cart or glider from rest and you may remove the spring or rubber band used to launch the cart or glider at constant speed. Release the cart or glider by removing a pencil or ruler from in front of the cart or glider – this minimizes error.
13) For two different heights, measure the times required to reach each of the five marks, repeat each measurement three times, and record the data in Table 3.3.
14) For the data in Table 3.3, calculate the instantaneous velocity, v_i, time squared, $t^2 = z$, means, m, and standard deviations, s, of v_i and $t^2 = z$, and record in Table 3.4.
15) Make two graphs with three curves on each graph. On one graph plot v_i vs. t for the three different elevations (one curve per elevation). On the other graph plot d vs. $t^2 = z$ for the three different elevations (one curve per elevation). Make these graphs on a separate page, attach to lab report, and reference this item number.

16) Draw best fit lines through these curves, find the slopes, calculate |g|, and tabulate in Table 3.5. Do you expect the best fit lines to intercept zero? Explain. Explain why we asked you to plot the distance, d, on the vertical axis and time squared, $z = t^2$ on the horizontal axis? Attach separate page to lab report and reference this item number.

Post-Lab Questions – Answer on a separate page and attach

17) In the free fall plane experiment you just completed, how close were your values to the accepted values? Explain and discuss sources of random and systematic error. Attach separate page to lab report and reference this item number.
18) In the inclined plane experiment you just completed, how close were your values to the accepted values? Explain and discuss sources of random and systematic error. Attach separate page to lab report and reference this item number.
19) You were asked to plot graphs in two different manners, v_i vs. t and d vs. $z = t^2$. Did one method give superior results? Or did the two techniques give about the same results with equivalent errors? Explain why one method may be superior to the other or, alternatively, why the methods should be equivalent. Attach separate page to lab report and reference this item number.
20) Compare the values of g obtained from free fall compared to the inclined plane? Which method do you feel provides the most precise and/or accurate value of g? Explain on a separate page.
21) Was it possible to calculate |g| from the level air track? What did the v_i vs. t graph show for the level track? What did the d vs. $z = t^2$ graph show for the level track? Were any of the results invalid or based on erroneous assumptions? If any results were invalid, explain why. Attach separate page to lab report and reference this item number.
22) What is the maximum acceleration possible for the v_i vs. t graph? Under what conditions would you achieve this acceleration? Attach separate page to lab report and reference this item number.

Lab 3 Report: Names ____

____

Equations:

How far does an object falls in a gravitational field in a given time interval t. Write that equation on the line below including the effect of a non-zero initial velocity, v_o:

____ ***Equation 3.6***

Now write this equation on the line below for the case of zero initial velocity:

____ ***Equation 3.7***

Equation of acceleration given $|g|$ and θ.

____ ***Equation 3.8***

Equation for acceleration down the plane given x, y, h and $|g|$. Write that equation below:

____ ***Equation 3.9***

Equation for instantaneous velocity, v_i, vs. distance down the plane, d_i, and, t_i. ($v_o = 0$).

____ ***Equation 3.10***

Table 3.1 – Table of times of fall, t_f (   )

	Trial	y_1 = _______( )	y_2 = _______( )	y_3 = _______( )
m_1 = ______ ( )	1			
	2			
	3			
m_2 = ______ ( )	1			
	2			
	3			
m_3 = ______ ( )	1			
	2			
	3			

Table 3.2 – Table of $|g|$ values (   )

	Trial	y_1 = ________( )	y_2 = ________( )	y_3 = ________( )
m_1 =	1			
	2			
	3			
______	m_g			
( )	s_g			
	%Dev			
m_2 =	1			
	2			
	3			
______	m_g			
( )	s_g			
	%Dev			
m_3 =	1			
	2			
	3			
______	m_g			
( )	s_g			
	%Dev			

Table 3.3 – Table of times, t (   ), to travel different distances, d, at different elevations, y.

Record hypotenuse length here, h = ____________(   )

	Trial	Distance traveled by cart or glider along track, units = ________________				
		d_1 = ________	d_2 = ________	d_3 = ________	d_4 = ________	d_5 = ________
Level Track	1					
	2					
	3					
y_1 = ______ ( )	1					
	2					
	3					
y_2 = ______ ( )	1					
	2					
	3					

Table 3.4 – Table of instantaneous velocity, v_i, and times squared, $z = t^2$, (   ), to travel different distances, *d*, at different elevations, *y*.

Record: h = ____________(   ), units of d = (   ), units of v_i = (   ), units of z = (   )

		d_1 = ________		d_2 = ________		d_3 = ________		d_4 = ________		d_5 = ________	
	Trial	v_i	z	v_i	z	v_i	z	v_i	z	v_i	z
Level Track	1										
	2										
	3										
	m										
	s										
y_1 = ______ ( )	1										
	2										
	3										
	m										
	s										
y_2 = ______ ( )	1										
	2										
	3										
	m										
	s										

Table 3.5 – Table of slopes and $|g|$.

	Slope v_i vs. t ( )	$\|g\|$ from v_i vs. t ( )	%Error from v_i vs. t	Slope d vs. $z = t^2$ ( )	$\|g\|$ from d vs. $z = t^2$ ( )	%Error from d vs. $z = t^2$
Level Track						
y_1 = ________ ( )						
y_2 = ________ ( )						