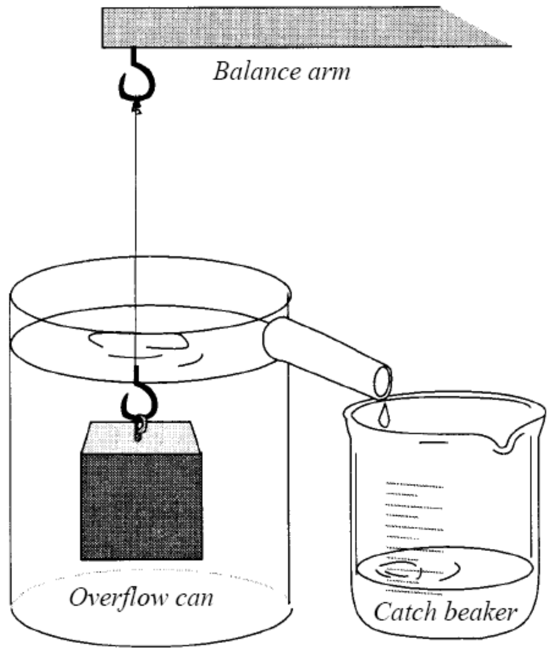
by J. T. Sizemore, P. Johnson, M. Brooks, G. Sherman, M. Broyles, A. Kumar

Archimedes' Principle



Purpose

You will use Archimedes' principle in this experiment to determine the densities of several solids and liquids.

Equipment

- Spring scales calibrated in N (preferred), force sensor, or mass balance rigged so object can be tied to it and lowered into water. There is a procedure to place the mass balance on the table, place a beaker of fluid on it, raise or lower objects into the fluid, and measure forces, however it requires a major revision of this theory and procedure and is therefore outside the scope of this laboratory.
- Lab jack, scissor jack, blocks, pieces of 2x4, or other blocks to raise and lower beakers, etc.
- Overflow can
- If mass balance is used, be sure to convert mass reading to force readings.
- 500-mL beaker
- 1000-mL beaker
- Large ringstand, stand with 90° clamps and crossbar, or other arrangement to hang objects being measured.
- Metal cube with hook. If it doesn't have a hook, tie it with string like a Christmas package.
- Lab balance
- String
- Wooden object with hook. If it doesn't have a hook, tie it with string like a Christmas package.
- Hydrometer or hydrometers capable of measuring liquids heavier and lighter than water
- Metal cylinder with hook. If it doesn't have a hook, tie it with string like a Christmas package.
- Bottle of unknown liquid such as saltwater, ethanol/water mixture, antifreeze/water mixture, etc.

Introduction

A solid object may float or sink when placed in a given fluid (i.e., in a gas or a liquid). If the object floats, it is buoyed up by a force equal to its weight. According to the ancient Greek philosopher Archimedes, the buoyant (upward) force exerted on an object that is either wholly or partially submerged in a fluid is equal to the weight of the amount of fluid displaced by the object. The object will sink when its weight exceeds the weight of the displaced fluid.

Because of Earth's gravity, every fluid substance from the atmosphere to the oceans to a tank full of gasoline has an internal pressure that increases with depth. And because gases are compressible, the greater pressure at a given depth squeezes the fluid into greater density at that depth. The density of an ideal gas is proportional to absolute pressure and inversely proportional to absolute temperature. Liquids are much less compressible, however the density of water increases about 0.3% as you descend to the bottom of the ocean.

We live our lives at the bottom of the atmospheric ocean which exerts about 101,300 N of force on every square meter of our bodies (10.13 N/cm^2). We don't notice this pressure, of course, because the fluids inside our bodies exert a balancing outward pressure. This balance is not automatic. Sometimes driving down a mountain the tube leading to our inner ear is temporarily blocked and we feel this pressure until we yawn to open the tube to "pop" our ears. The pressure change may be great enough that we cannot breathe normally and we must surround our bodies with air at normal atmospheric pressure if we ascend into the atmosphere or descend into the ocean.

Because of the vertical pressure gradient in fluids, the pressure on the lower surface of a submerged object is always greater than that on the upper surface. This is the root cause of the buoyant force.

Specific gravity is really nothing but relative density; that is, it is a unit-less ratio between the density of an object or a substance relative to the density of pure water, with both values of density expressed in the same units. Specific gravity is much more useful when dealing with different systems of units. The specific gravity of pure water is always 1.0, however the density is 1.0 gm/cm^3, 1000 kg/m^3, or 62.4 lb/ft^3. Thus the density of steel is 7.8 gm/cm^3, 7800 kg/m^3, or 487 lb/ft^3, but the specific gravity is always 7.8 independent of the system of units.

You will use Archimedes' principle in this experiment to determine the densities of several solids and liquids. When you finish the experiment, you will be able to:

1) Determine whether or not an object will float or sink in a fluid if you know the density of each.
2) Explain the difference between density and specific gravity.
3) Determine the density or specific gravity of a solid or fluid whether it floats or sinks.

Theory

A. Objects Denser Than Water

The buoyant force is described by Archimedes' principle as: *an object, when placed in a fluid, is buoyed up by a force equal to the weight of the fluid displaced by the object.* The principle applies to an object either entirely or partially submerged in the fluid. The magnitude of the buoyant force depends *only* on the weight of the displaced fluid, and not on the object's weight. Using Archimedes' principle, you can deduce that an object:

1) Will float in a fluid if the object's density is less than the fluid's density ($\rho_o < \rho_f$).
2) Will sink if the object's density is greater than the fluid's density ($\rho_o > \rho_f$).
3) Will remain in equilibrium at a given submerged depth if the object's density is exactly equal to the fluid's density at that depth ($\rho_o = \rho_f$).

The buoyant force on a floating object F_b is related to the properties of the displaced fluid by:

$$F_b = m_f g = \rho_f V_o g \qquad \textbf{\textit{Equation 11.1}}$$

where ρ_f is the density of the fluid, V_o is the volume of the submerged part of the object, and g is the acceleration due to gravity.

When working with forces it is important for the student to be able to draw a free body diagram and working with buoyant force is no exception. Figure 1 following shows the free body diagrams (a) in air and (b) in water.

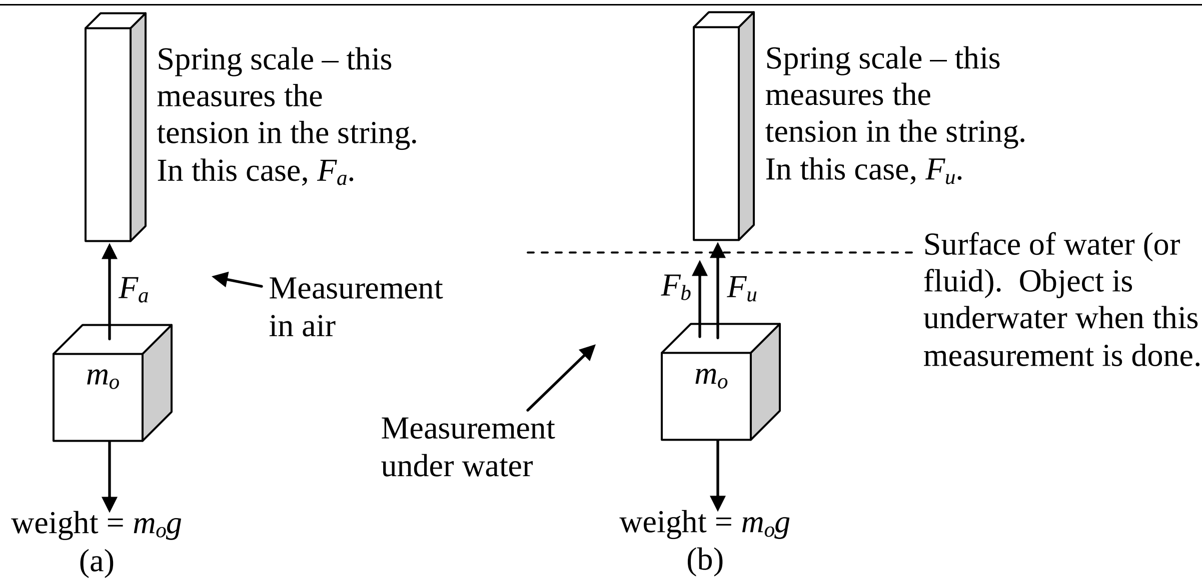


Figure 11.1 – Free body diagrams of forces on object of mass m_o. Neither the mass nor the weight of the object changes, however on the right hand side depicting the case of measurement underwater (or under some other fluid) there is a buoyant force, F_b. The spring scale (or force sensor, etc.) simply measure the tension in the string attached to the object. On the left hand side, measurement in air, this tension is equal and opposite to the weight (since the object is at rest). Note the density of air is quite small and we therefore presume the buoyant force of air on the object is negligible. On the right hand side, measurement of the object while it is under the surface of a fluid, the spring scale reads F_u – equal and opposite to the difference between the weight and buoyant force.

The volume of the submerged object oriented vertically is equal to its cross-sectional area A multiplied by the height h of the submerged part. Note that the force of the fluid on the top and bottom equals:

$$F_{bottom} = AP_{bottom} \text{ and } F_{top} = AP_{top} \qquad \textbf{\textit{Equation 11.2}}$$

Remember that the pressure on the bottom is slightly greater than the pressure on the top and the difference equals:

$$P_{bottom} = P_{top} + \rho_f hg \qquad \textbf{\textit{Equation 11.3}}$$

The buoyant force is $F_{bottom} - F_{top}$. Using this and rearranging Equations 11.2 and 11.3 tells us the buoyant force:

$$F_b = \rho_f Agh \qquad \textbf{\textit{Equation 11.4}}$$

Since $Ah = V_o$ it's easy to observe Equation 11.4 is equivalent to Equation 11.1. This is a linear relationship between F_b and h, so if you lower the cylinder into a fluid as you measure its weight, then plot F_b vs. h, the slope of the plotted straight line will be $\rho_f Ag$, that is, directly proportional to the density of the fluid. This is a cool way to determine the density of an unknown fluid.

You can determine the density of an unknown solid object in a similar fashion. It's easy to measure the mass of an object, but unless it has a regular shape it's not so easy to measure its volume. But Archimedes showed us how to measure volume by measuring weight.

When the object is completely submerged in a fluid, the measured force (but not its mass) will decrease by an amount equal to the upward buoyant force the water exerts on it (refer to Figure 11.1). So:

$$\Delta F_o = F_a - F_u = F_b \qquad \textbf{\textit{Equation 11.5}}$$

Where $F_a = m_o g$ is the weight in air and F_u is the weight when submerged underwater. This upward force is also equal to the weight of the displaced fluid, or:

$$\Delta F_o = W_f = F_b = m_f g = \rho_f g V_f \qquad \textbf{\textit{Equation 11.6}}$$

What Equation 11.6 is telling us is that the buoyant force, F_b, is the weight of the fluid displaced, W_f. If we set up the overflow can as in Figure 11.2 (as follows), put in water until is overflows (and discard excess water), then drop an object into the overflow can, collect the water that overflows, and weigh the overflow THEN the weight of the overflow will equal the buoyant force.

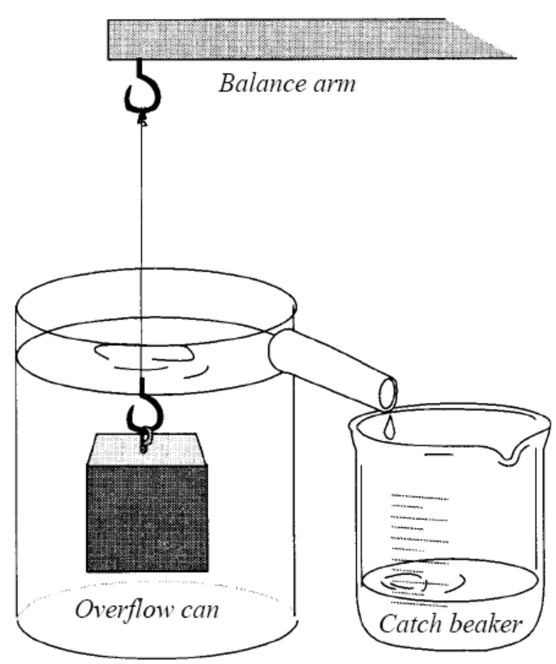


Figure 11.2 – Experimental arrangement to catch overflow and weigh the overflow water. This weight should equal the buoyant force.

We have a second way to measure F_b per Equations 11.5 and 11.6. We weigh the object in air, F_a, and subtract the weight when submerged underwater, F_u. We will compare the two methods. The volume of the water is equal to the volume of the object, so:

$$V_f = V_o = \frac{\Delta F_o}{\rho_f g} \qquad \textbf{\textit{Equation 11.7}}$$

The density of the object is therefore (recall F_a is the force reading in air and equals the weight of the object):

$$\rho_o = \frac{m_o}{V_o} = \frac{m_o \rho_f g}{\Delta F_o} = \rho_f \left(\frac{F_a}{F_a - F_u} \right) \qquad \textit{Equation 11.8}$$

B. *Density of an Unknown Liquid*

You can also determine the density of an unknown liquid without measuring the submerged height of the solid object. Recall Equations 11.4 and 11.5 and Figure 11.1. When we measure a sinking object in water we obtain $F_{bw} = \rho_w V_o g = F_a - F_{uw}$ where ρ_w refers to the density of water and when we measure the same object in the unknown fluid we obtain $F_{bf} = \rho_f V_o g = F_a - F_{uf}$. Where the subscript "b" refers to buoyant force, subscript "w" refers to water, subscript "f" refers to unknown fluid, subscript "a" refers to measurements in air, and subscript "u" refers to submersion under the water or unknown fluid. The ratio of F_{bf} to F_{bw} is given by the following equation:

$$S.G. = \frac{\rho_f}{\rho_w} = \frac{F_{bf}}{F_{bw}} = \frac{F_a - F_{uf}}{F_a - F_{uw}} \qquad \textit{Equation 11.9}$$

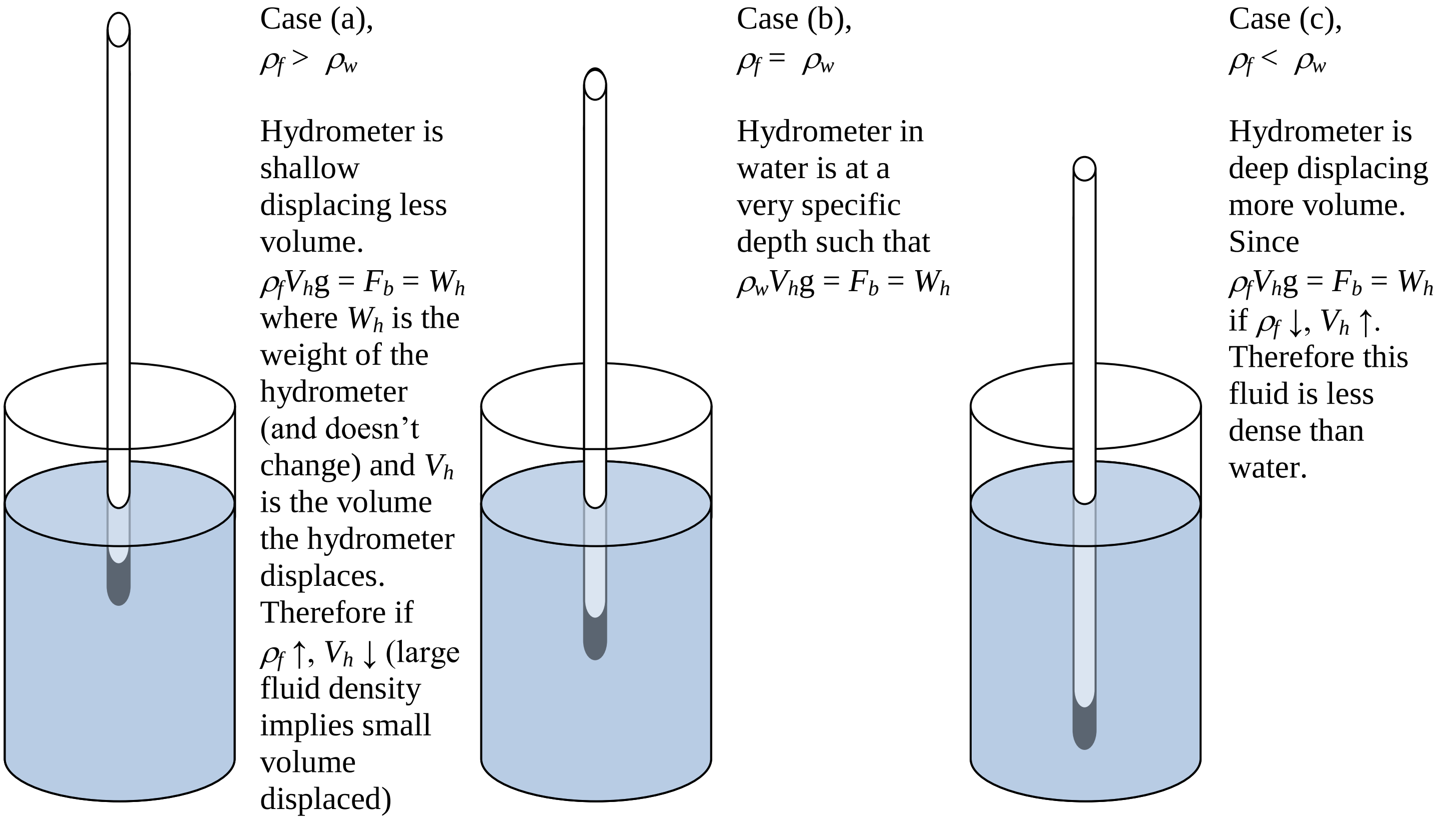


Figure 11.3 – Hydrometer in three different density liquids. Hydrometer is shallow if fluid is more dense than water, hydrometer is medium if fluid density equals water, and hydrometer is deep if fluid is less dense than water.

Multiplying the specific gravity of the unknown fluid given by Equation 11.9 by the density of water gives the density of the unknown fluid.

$$\rho_f = \rho_w (S.G.) = \rho_w \frac{F_{bf}}{F_{bw}} = \rho_w \left(\frac{F_a - F_{uf}}{F_a - F_{uw}}\right) \qquad \textbf{\textit{Equation 11.10}}$$

Devices to measure S.G. are known as hydrometers. They are simply closed tubes weighted at the bottom of known mass. The deeper in the fluid the hydrometer goes, more volume of fluid is displaced raising the buoyant force. When the buoyant force equals the weight, the hydrometer stops. For a fluid denser than water the hydrometer rides higher (less deep) than for water. For a fluid less dense than water the hydrometer goes deeper. A scale on the side of the hydrometer is calibrated to indicate specific gravity. This is illustrated in Figure 11.3 shown previously.

C. Objects Less Dense Than Water

Most hydrometers are less dense than water and that gives us a hint of how to measure the density of a liquid less dense than water. We will explore two methods – the first being the principle the hydrometer uses. For the most part it's impractical – objects have to be shaped like hydrometers and most are not. Therefore we need a second method – attach a sinker to a floating object to make it sink and measure the forces. In the following section we will develop the theory behind both.

The first question is, for an object less dense than water, what fraction of it is above the surface? When an object floats the buoyant force is equal and opposite to the weight. Therefore:

$$F_b = \rho_f V_u g = weight = \rho_o V_o g \qquad \textbf{\textit{Equation 11.11}}$$

Where V_u is the volume of the object underwater and V_o is the total volume of the object. As previously used, ρ_f is the density of the fluid, ρ_o is the density of the object, and g is the acceleration of gravity. Since $V_o - V_u$ is the volume above water, by rearranging Equation 11.11 we find the fraction above water is:

$$\frac{V_o - V_u}{V_o} = 1 - \left(\frac{V_u}{V_o}\right) = 1 - \left(\frac{\rho_o}{\rho_u}\right) = \text{fraction above water} \qquad \textbf{\textit{Equation 11.12}}$$

Typically 60% of a wooden object is above water (however it depends greatly on the type of wood). If this is true, what is the density of this wood? We use Equation 11.12 to find the answer and it is $\rho_o = 0.4$ gm/cm^3.

Applying Equation 11.12 is usually impractical due to the irregular shape of ordinary objects including icebergs. We can, however, tie a sinker to a floating object and measure forces to find the object's density. Let's first look at the free body diagrams in Figure 11.4 below.

F_a is the spring scale reading in air for the object and sinker, F_s is the spring scale reading when only the sinker is underwater, and F_u is the spring scale reading when both are underwater. The buoyant force acting on the object is simply $F_{bo} = F_s - F_u = \rho_f V_o g$. We can then proceed just like Equations 11.7 and 11.8:

$$V_f = V_o = \frac{F_{bo}}{\rho_f g} \qquad \textbf{\textit{Equation 11.13}}$$

The density of the object is therefore (recall F_o is the weight of the object, force of gravity on the object, and equals $m_o g$):

$$\rho_o = \frac{m_o}{V_o} = \frac{m_o \rho_f g}{F_{bo}} = \rho_f \left(\frac{m_o g}{F_s - F_u}\right) = \rho_f \left(\frac{F_o}{F_s - F_u}\right) \qquad \textbf{\textit{Equation 11.14}}$$

See! This isn't very hard. Just keep the free body diagrams in mind, figure out forces, and calculate density. Also choose variable that remind you of what they physically mean. You'll notice that the subscript "s" refers to a sunken sinker, "u" refers to both underwater, F_{bo} is the buoyant force on the object, etc. Naming things well is a mnemonic device to help us remember their meaning.

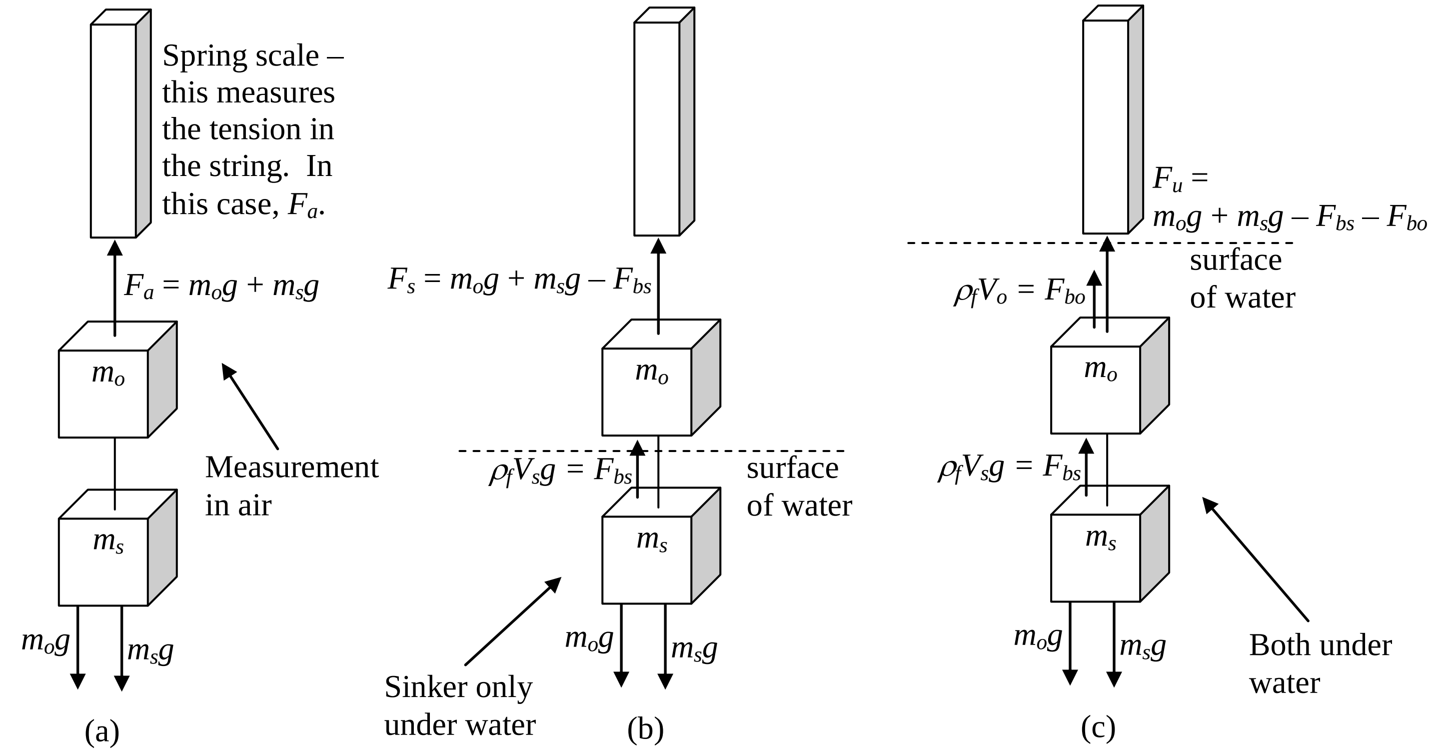


Figure 11.4 – Free body diagrams of forces on object of mass m_o and sinker of mass m_s. when measured in air, only the sinker is submerged, and when both are submerged.

<u>Procedure</u>

You will:

1. Weigh a sinking object in air and under water
2. Weigh the water displaced
3. Weigh a floating object in air and under water (using a sinker)
4. Measure the volume of the floating object, calculate density directly, and compare results with results from using Archimedes' Principle
5. Weigh the sinking object under water and under an unknown liquid
6. Find the specific gravity of an unknown liquid using a hydrometer

From these measurements, you will calculate the densities of the objects and liquids. It is preferable to use spring scales to measure forces directly, however if mass balances are used be sure to convert mass readings to force readings.

A. <u>Density of a Sinking Object</u>

1. Place the spring scale on the ringstand. Alternatively, if using a lab balance, mount the lab balance on a ringstand about two feet high.
2. Using thread tie the object with a loop on the end of the thread.

3. Place the overflow can on a lab jack under the object.
4. Weigh the metal object m_o , calculate its weight, F_a (force reading in air), and record this in Table 11.1.
5. Weigh the dry catch beaker m_b and record its mass in Table 11.1.
6. Place the catch beaker beneath the spout of the overflow can as shown in Figure 11.2, and fill the overflow can with water until it runs out the spout. Then empty and replace the beaker.
7. Hang the metal object by the short thread from the hook beneath the balance pan, then raise the lab jack until the object is completely submerged in the water. Weigh the submerged object and record its force, F_u, in Table 11.1.
8. Remove the sample from the hook under the scale or balance.
9. Measure and record the combined mass of the catch beaker and the overflow water m_{comb}.
10. Record the measured mass m_w of the displaced water.
11. Calculate the weight of the displaced water, W_w, and record this in Table 11.2.
12. Use Equation 11.5 to calculate the buoyant force, F_b, and record this in Table 11.2.
13. Calculate and record the percent difference between W_w and F_b. in Table 11.2
14. Using Equation 11.8, calculate and record the density, ρ_o, of the object and record this in Table 11.2.
15. Record the accepted value of density (consult your textbook for this value), ρ_{acc}, and record this in Table 11.2.
16. Calculate the percent error between measured density, ρ_o, and the accepted value, ρ_{acc}, and record this in Table 11.2.

B. *Density of a Liquid*

17. Use the same object from Part A and, since this was performed in water, the force in air, F_a, in Table 11.1 and the buoyant force, F_b, in Table 11.2 will be needed. Transfer this information to Table 11.3 noting that $F_{bw} = F_b$ from Table 11.2.
18. Replace the overflow can with the unknown liquid in a beaker. Repeat the procedure in step 7 and record the force, F_{uf}.
19. Use Equations 11.9 and 11.10 to calculate the density, ρ_f, and specific gravity, *S.G.*, of the unknown liquid and record these values in Table 11.3.
20. Use the hydrometer to measure the specific gravity of the unknown liquid and record this as $S.G._h$.
21. Using $S.G._h$ compute the density and record it as ρ_h.
22. Find the percent difference between ρ_f and ρ_h.

C. *Density of a Floating Object*

23. The procedure for this is similar to Part A, but we won't do overflow. Instead use a regularly shaped floating object (rectangular solid, cylinder, etc.). Using your knowledge of the volume of solids, measure and calculate the volume and record this in Table 11.4. Below Table 11.4 describe how you calculate this volume and record data taken to perform this calculation.
24. Place the spring scale on the ringstand. Alternatively, if using a lab balance, mount the lab balance on a ringstand about two feet high.
25. Weigh the sinker, m_s, and the floating object, m_o. From Equation 11.14 we observe that it is not necessary to know the mass of the sinker, but just-in-case and for good measure, we'll record it anyway. Record m_o, calculate the object's weight, F_o, and record it in Table 11.4.
26. From m_o and V_o calculate the density. Record it in the column ρ_{od} (<u>d</u>ensity <u>o</u>bject <u>d</u>irect measurement).
27. Using thread tie the sinker below the floating object and tie a loop above the floating object.
28. Place a beaker about half full of water on a lab jack under the sinker/object combination.
29. Also for good measure record the density of the fluid, ρ_f. Since you're using water, $\rho_f = 1.00$ gm/cm^3.
30. Record the weight (force in air), F_a, of the sinker/object combination.

31. Hang the sinker/object combination by the short thread from the hook beneath the balance pan (or spring scale), then raise the lab jack until the sinker ONLY is completely submerged in the water. Record this force, F_s.
32. Now raise the lab jack until both the sinker and object are completely submerged, BUT THE SINKER SHOULD NOT BE RESTING ON THE BOTTOM. If necessary, add more water. Record this force, F_u.
33. Use Equation 11.14 to calculate the density, ρ_o, of the object and record this.
34. Clean up and put away equipment.

Post-Lab Questions – Answer on a separate page and attach

1. In Table 11.2 you found the buoyant force using two methods. Do your results confirm or contradict Archimedes' Principle? Explain thoroughly.
2. Also in Table 11.2 you compared your measured density with the accepted density? Do you believe the Archimede's Principle is a good method or poor method to measure density? Explain thoroughly.
3. Due to the string used, will the measured force be greater than or less than the true force? Also due to the string, will the measured volume be greater than or less than the true volume? Combining these two influences will the measured density be greater than or less than the true density or about the same? Explain thoroughly.
4. As a hypothetical example, if you submerge an object with the same density as the fluid, will the object sink, rise, or remain unchanged? Explain.
5. How would you cause an un-tethered, at rest, un-powered submarine to rise? To sink? Are you changing the density, volume, and/or mass of the submarine? How is this different from a hot air balloon? Explain.
6. What is the net force on a floating ice cube? A floating block of wood? A steel bolt floating in mercury? Explain.
7. Which is heavier, 1 cm^3 of ice or 1 cm^3 of water? Explain.
8. If you have a beaker of water and a block of wood balanced on a mass balance and you put the wood in the water, will the mass balance change? Why or why not? Wouldn't the buoyant force on the wood cause the mass balance to show less mass when the wood is in water? Similarly, if pigeons in a truck were flying, would it weigh less? Explain.
9. A lift is rated to lift 10,000 N (about a ton). If it were lifting an aluminum object (ρ = 2.7 gm/cm^3) out of a lake, what is the maximum mass of aluminum it can lift? What is the weight of this maximum mass?
10. Explain how a hydrometer works. Consider one with a cross-sectional area of 1 cm^3 and mass of 100 gm. What length is underwater? If it measures alcohol with ρ = 0.8 gm/cm^3 how much is below the surface? If it measures antifreeze with ρ = 1.1 gm/cm^3 how much is below the surface? Assume it's sufficiently long that it won't sink.

Lab 3 Report: Names ____

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Table 11.1 – Raw Data for Density of Sinking Object and Direct Confirmation of Archimedes' Principle

Object Material	m_o ( )	F_a ( )	m_b ( )	F_u ( )	m_{comb} ( )	m_w ( )

Table 11.2 – Results for Density of Sinking Object and Direct Confirmation of Archimedes' Principle

Object Material	W_w ( )	F_b ( )	$F\ Err\%$ ( )		ρ_o ( )	ρ_{acc} ( )	$\rho\ Err\%$ ( )

Table 11.3 – Density of an Unknown Liquid

F_a ( )	F_{bw} ( )	F_{uf} ( )	ρ_f ( )	$S.G.$ ( )	$S.G._h$ ( )	ρ_h ( )	$\rho\ \Delta\%$ ( )

Table 11.4 – Density of a Floating Object

m_s = ( ), F_a = ( ), ρ_f = ( )

Object Material	V_o ( )	m_o ( )	ρ_{od} ( )	F_o ( )	F_s ( )	F_u ( )	ρ_o ( )	$\rho_o\ Err\ \%$ ( )